ABDUL LATIF JAMEEL
Poverty Action Lab

TRANSLATING RESEARCH INTO ACTION

Sampling and Sample Size

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Course Overview

1. What is evaluation?
2. Measuring impacts (outcomes, indicators)
3. Why randomize?
4. How to randomize
5. Threats and Analysis
6. Sampling and sample size
7. RCT: Start to Finish
8. Cost Effectiveness Analysis and Scaling Up

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1. What is evaluation?
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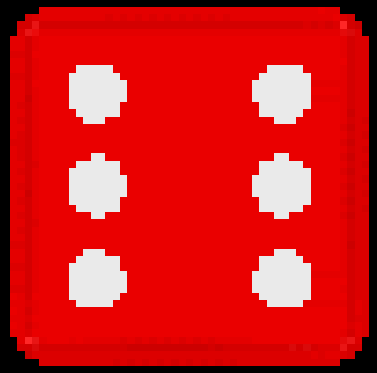
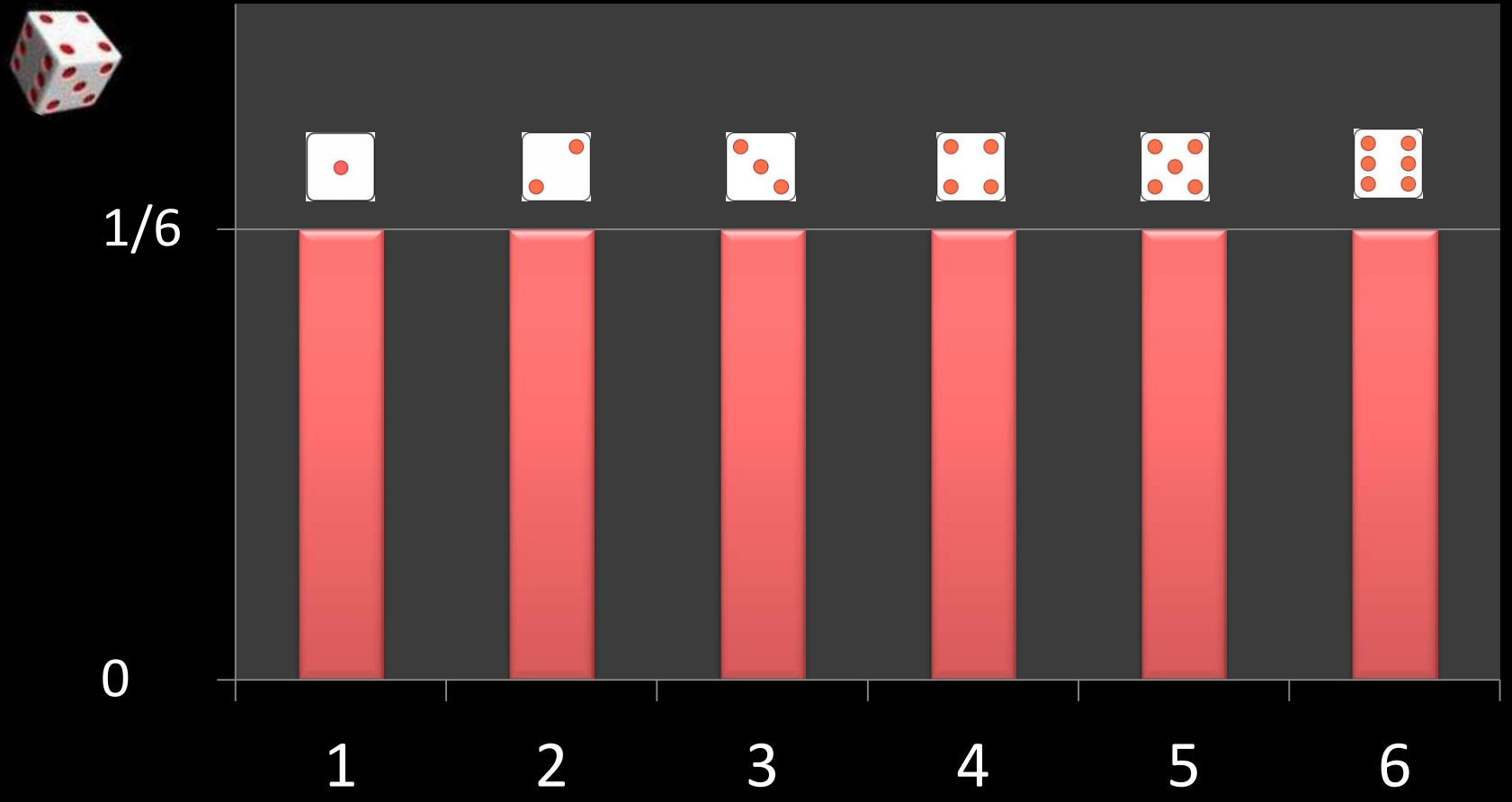
6. **Sampling and sample size**
7. RCT: Start to Finish
8. Cost Effectiveness Analysis and Scaling Up

What's the average result?

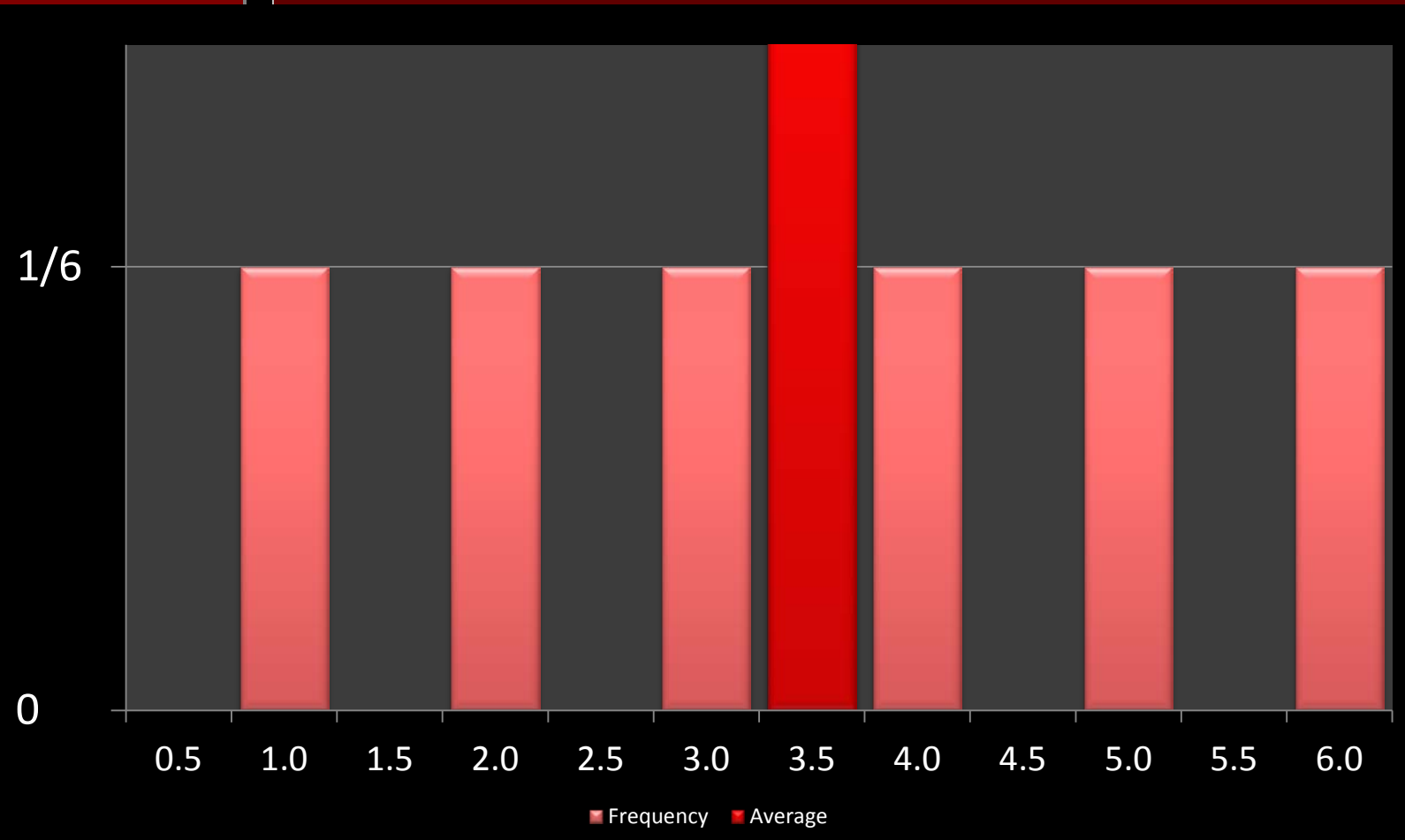
- If you were to roll a die once, what is the "expected result"? (i.e. the average)



Possible results & probability: 1 die



Rolling 1 die: possible results & average



What's the average result?

- If you were to roll two dice once, what is the expected average of the two dice?

Rolling 2 dice: Possible totals & likelihood

	2	3	4	5	6	7	8	9	10	11	12
Frequency	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Rolling 2 dice: possible totals
12 possible totals, 36 permutations

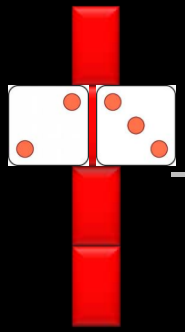
Die 2 \ Die 1	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Rolling 2 dice:
Average score of dice & likelihood

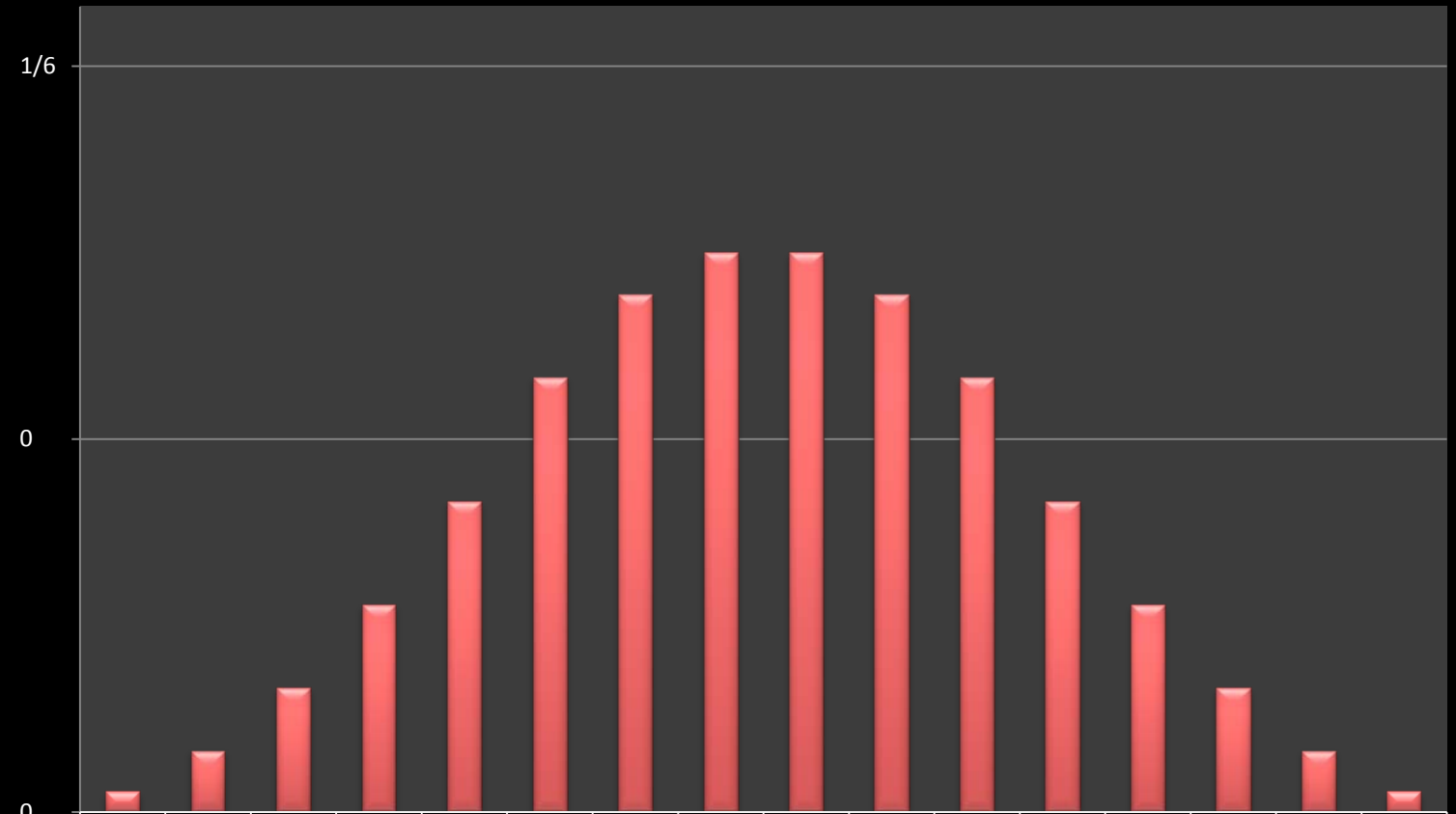
	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Frequency	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Outcomes and Permutations

- Putting together permutations, you get:
  1. All possible outcomes
  2. The likelihood of each of those outcomes
  - Each column represents one possible outcome (average result)
  - Each block within a column represents one possible permutation (to obtain that average)

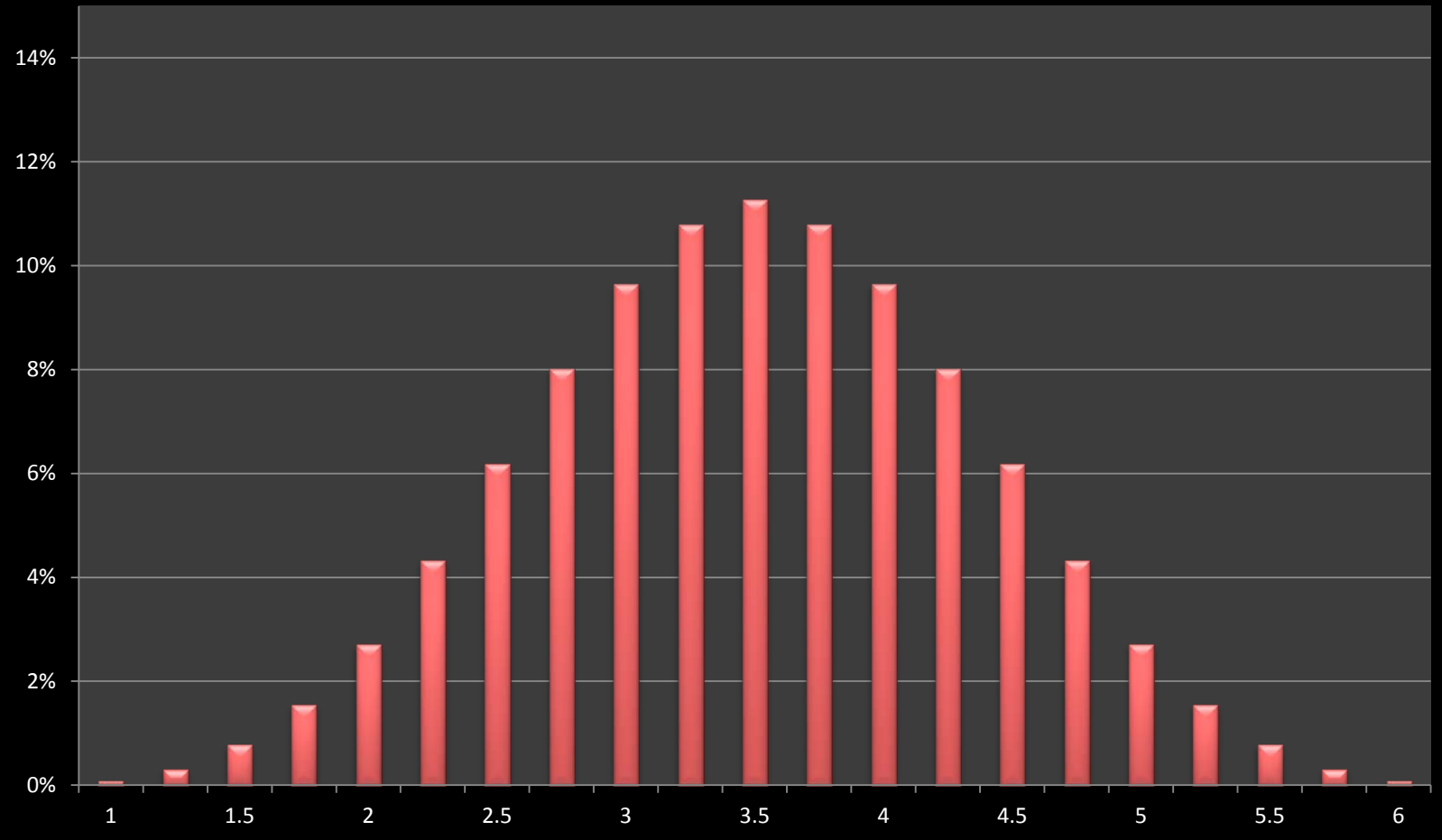


Rolling 3 dice:
16 results 3→18, 216 permutations

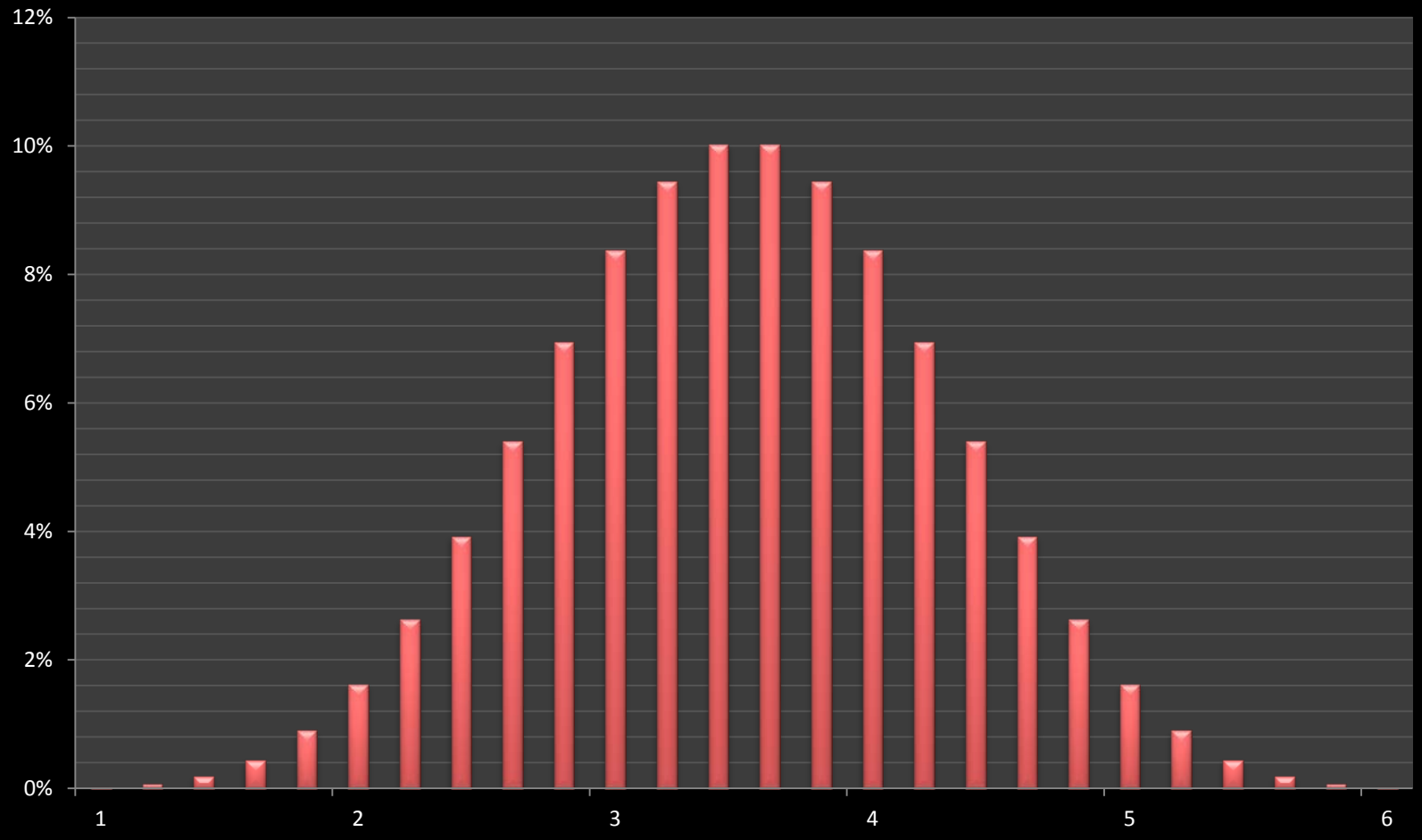


	1	1 1/3	1 2/3	2	2 1/3	2 2/3	3	3 1/3	3 2/3	4	4 1/3	4 2/3	5	5 1/3	5 2/3	6
Frequency	0%	1%	3%	5%	7%	10%	12%	13%	13%	12%	10%	7%	5%	3%	1%	0%

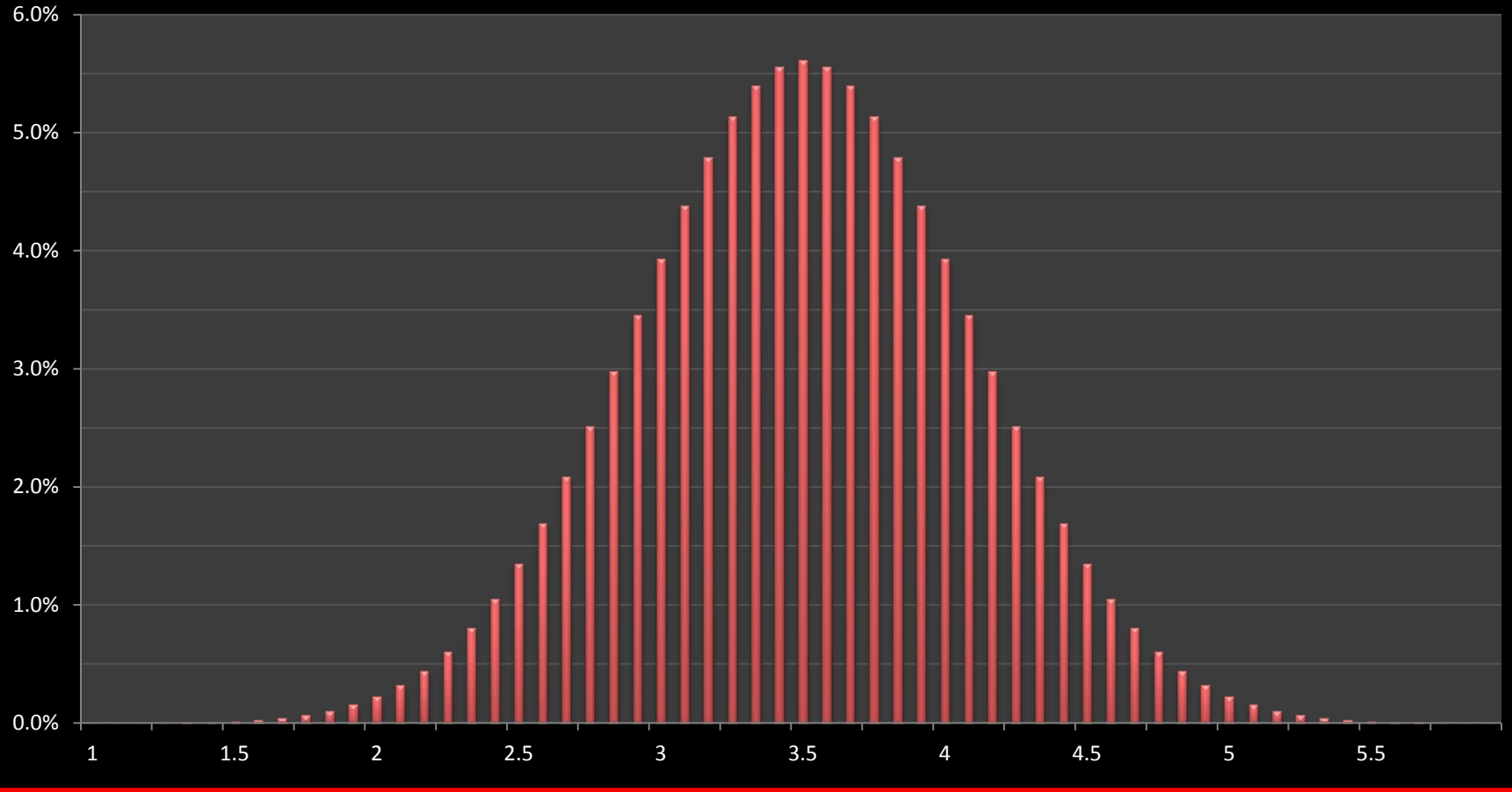
Rolling 4 dice:
21 results, 1296 permutations



Rolling 5 dice:
26 results, 7776 permutations

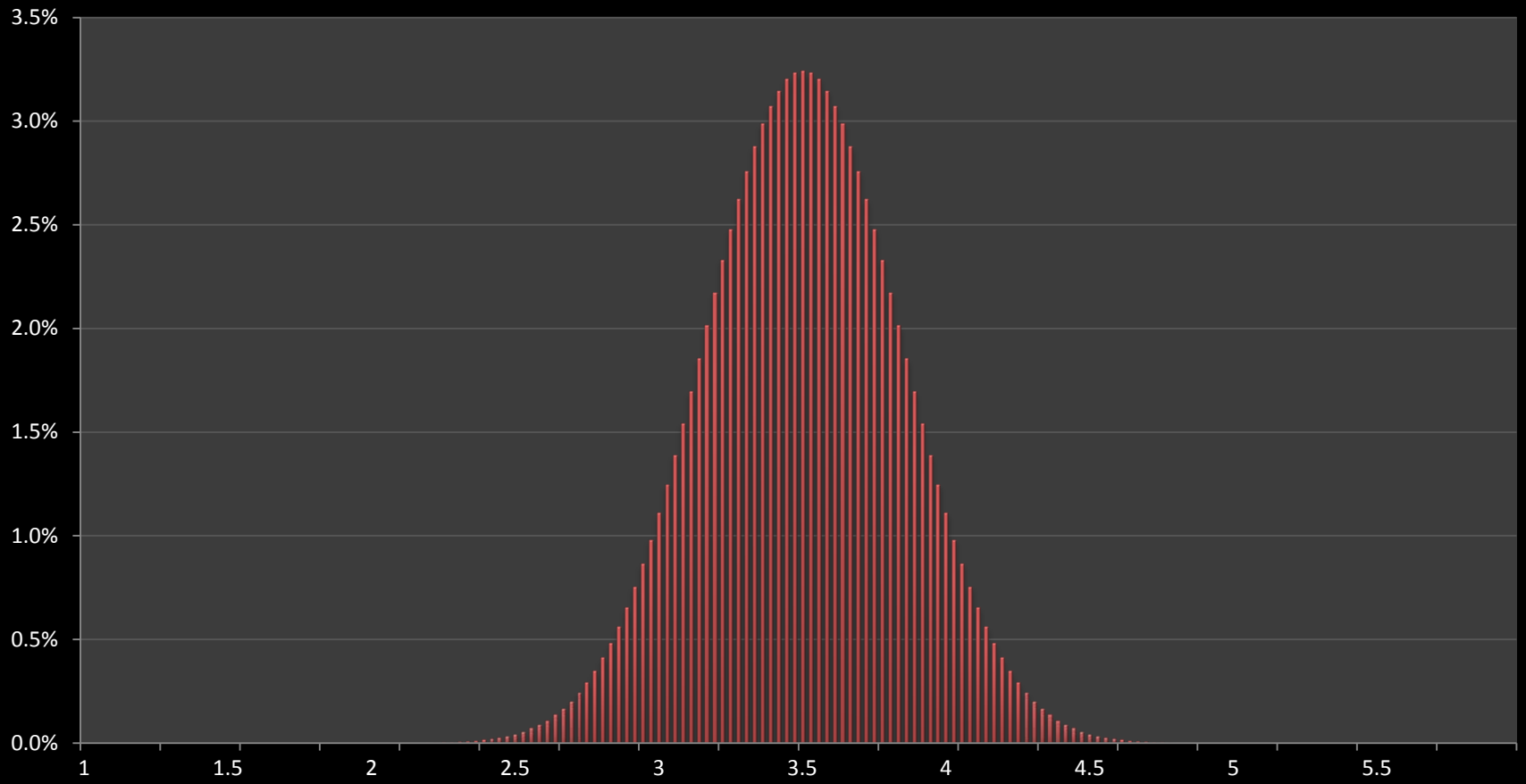


Rolling 10 dice:
50 results, >60 million permutations



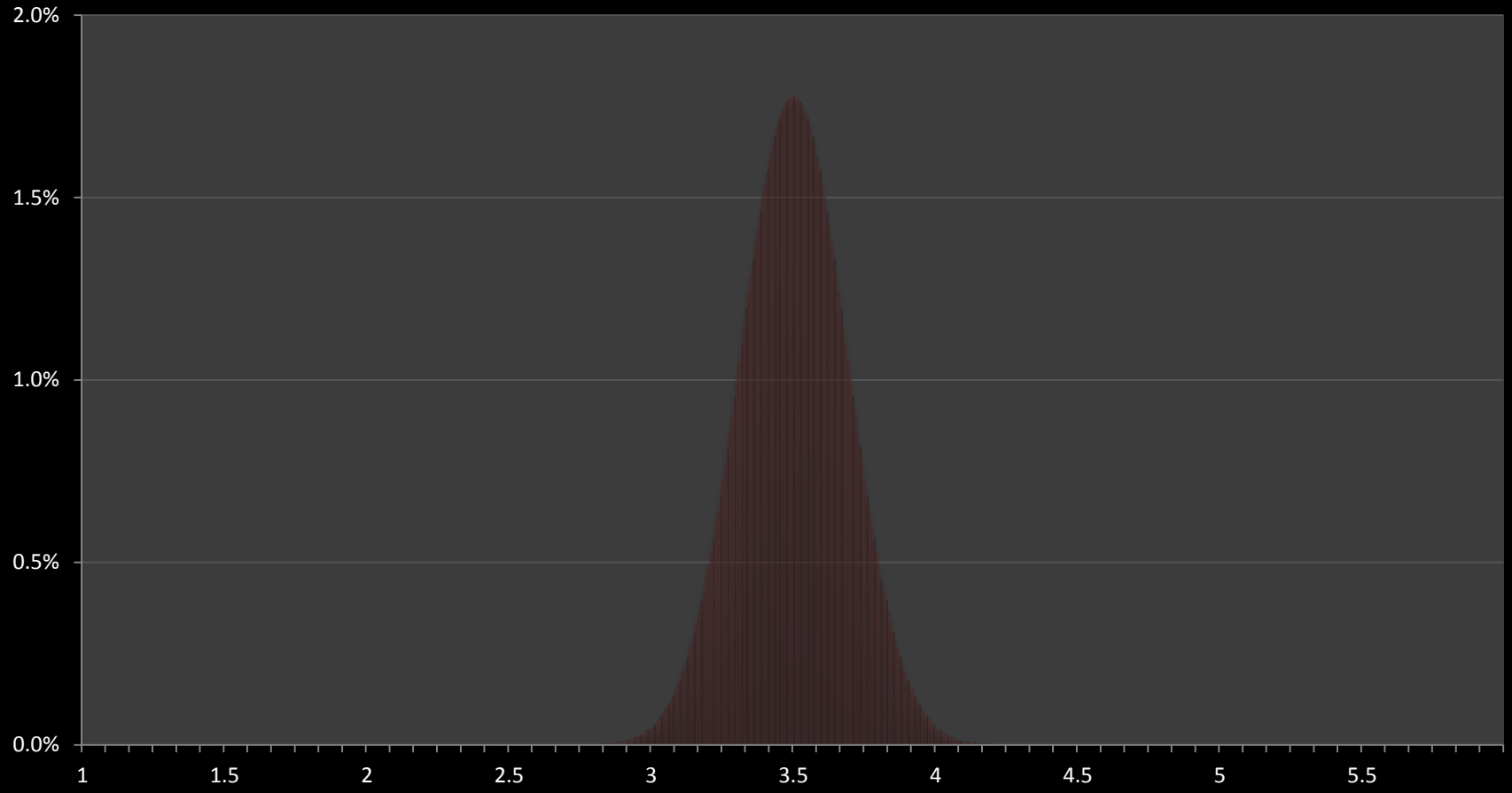
Looks like a bell curve, or a *normal* distribution

Rolling 30 dice:
150 results, 2×10^{23} permutations*



>95% of all rolls will yield an average between 3 and 4

Rolling 100 dice:
500 results, 6×10^{77} permutations



>99% of all rolls will yield an average between 3 and 4

Rolling dice: 2 lessons

1. The more dice you roll, the
   closer most averages are to the true average
   (the distribution gets "tighter")
   -THE LAW OF LARGE NUMBERS-
2. The more dice you roll, the
   more the distribution of possible averages
   (the *sampling distribution*)
   looks like a bell curve (a *normal* distribution)
   -THE CENTRAL LIMIT THEOREM-

Which of these is more accurate?

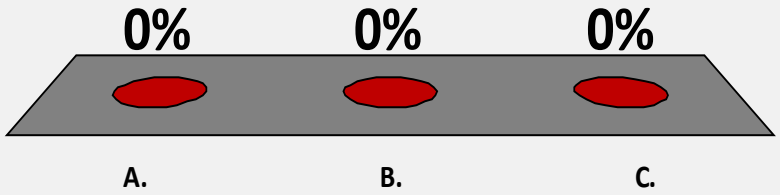
I. 

II. 

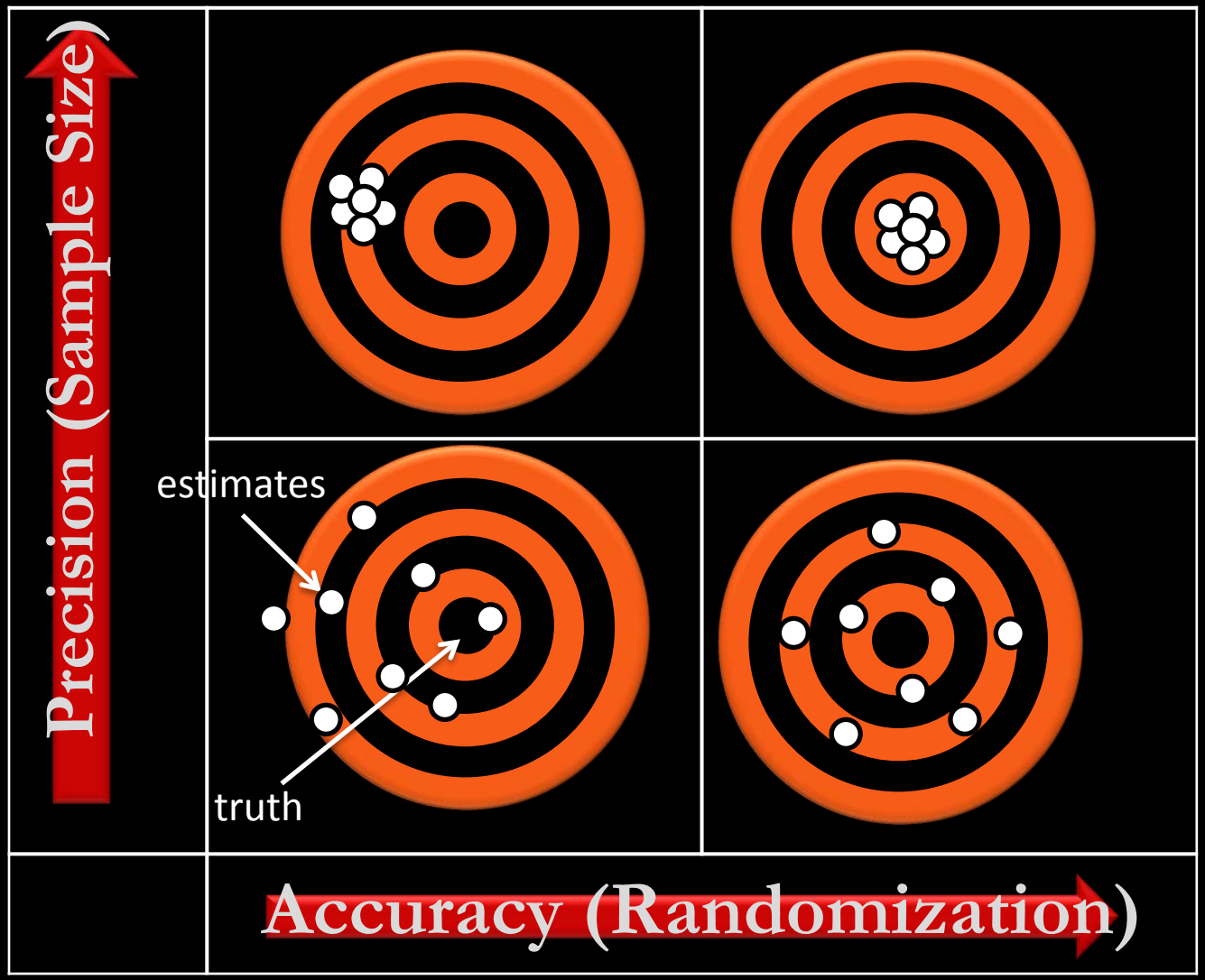
A. I.

B. II.

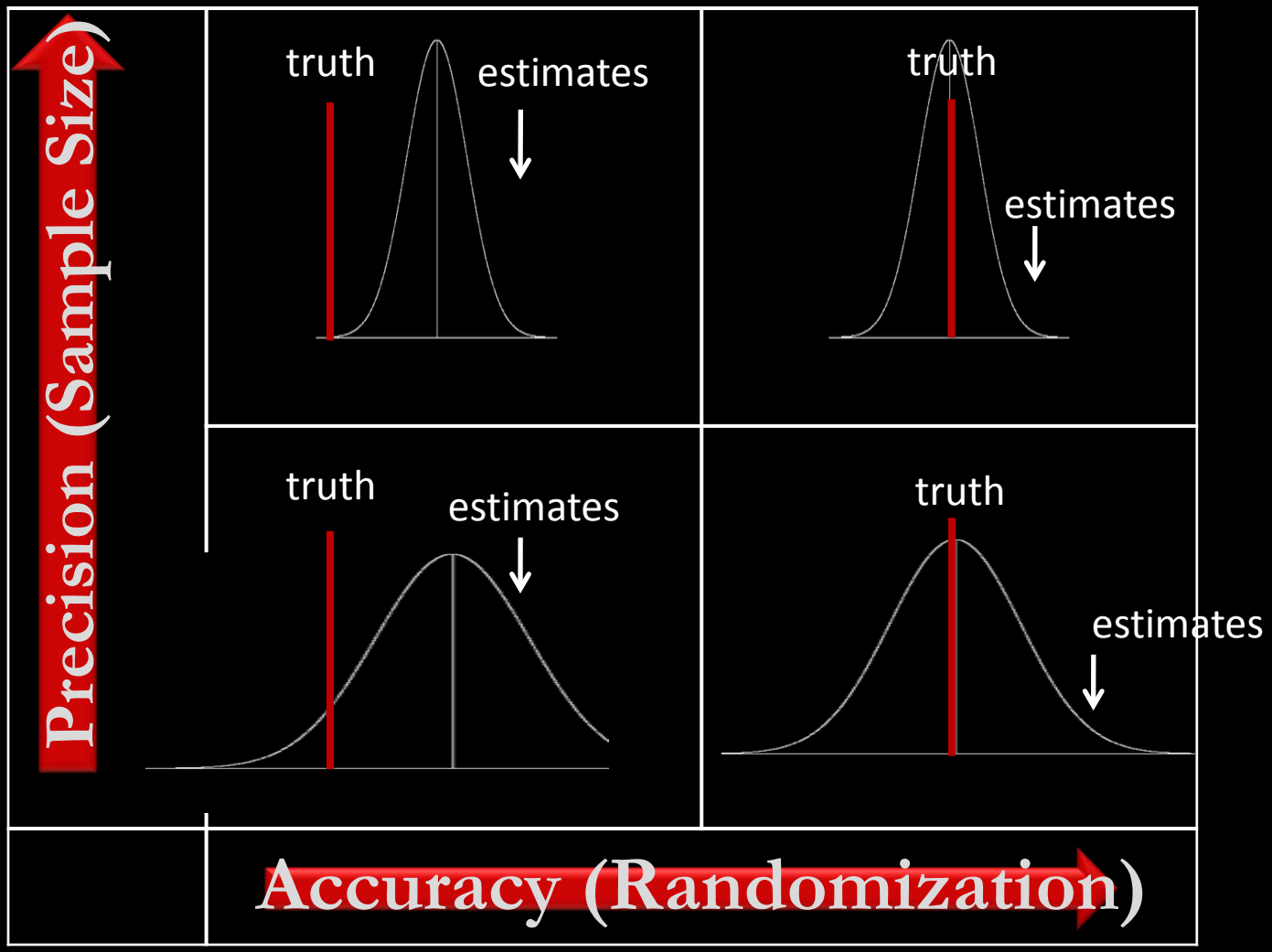
C. Don't know



Accuracy versus Precision



Accuracy versus Precision



THE basic questions in statistics

- How confident can you be in your results?

- → How big does your sample need to be?

THAT WAS JUST THE INTRODUCTION

Outline

- Sampling distributions
  - population distribution
  - sampling distribution
  - law of large numbers/central limit theorem
  - standard deviation and standard error
- Detecting impact

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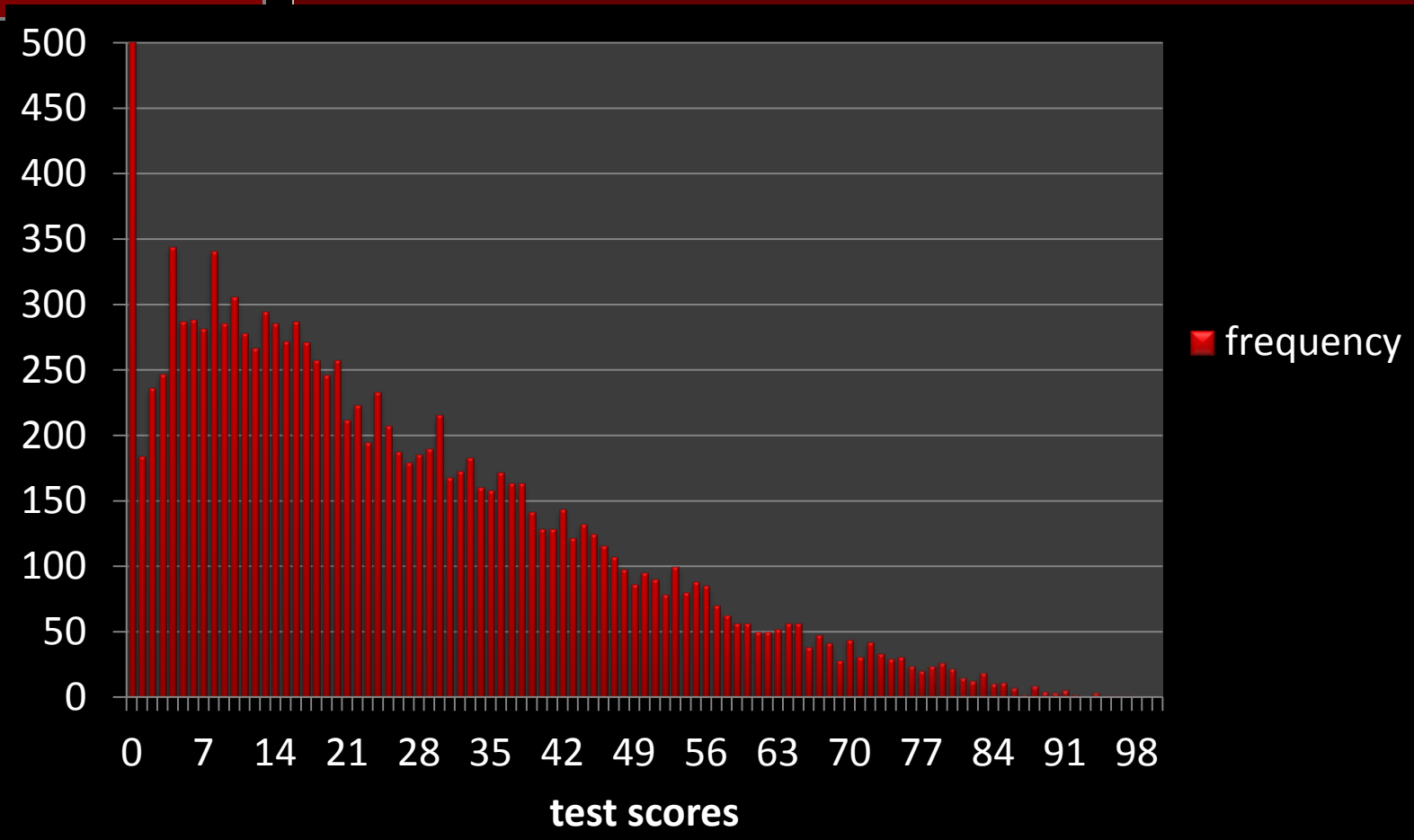
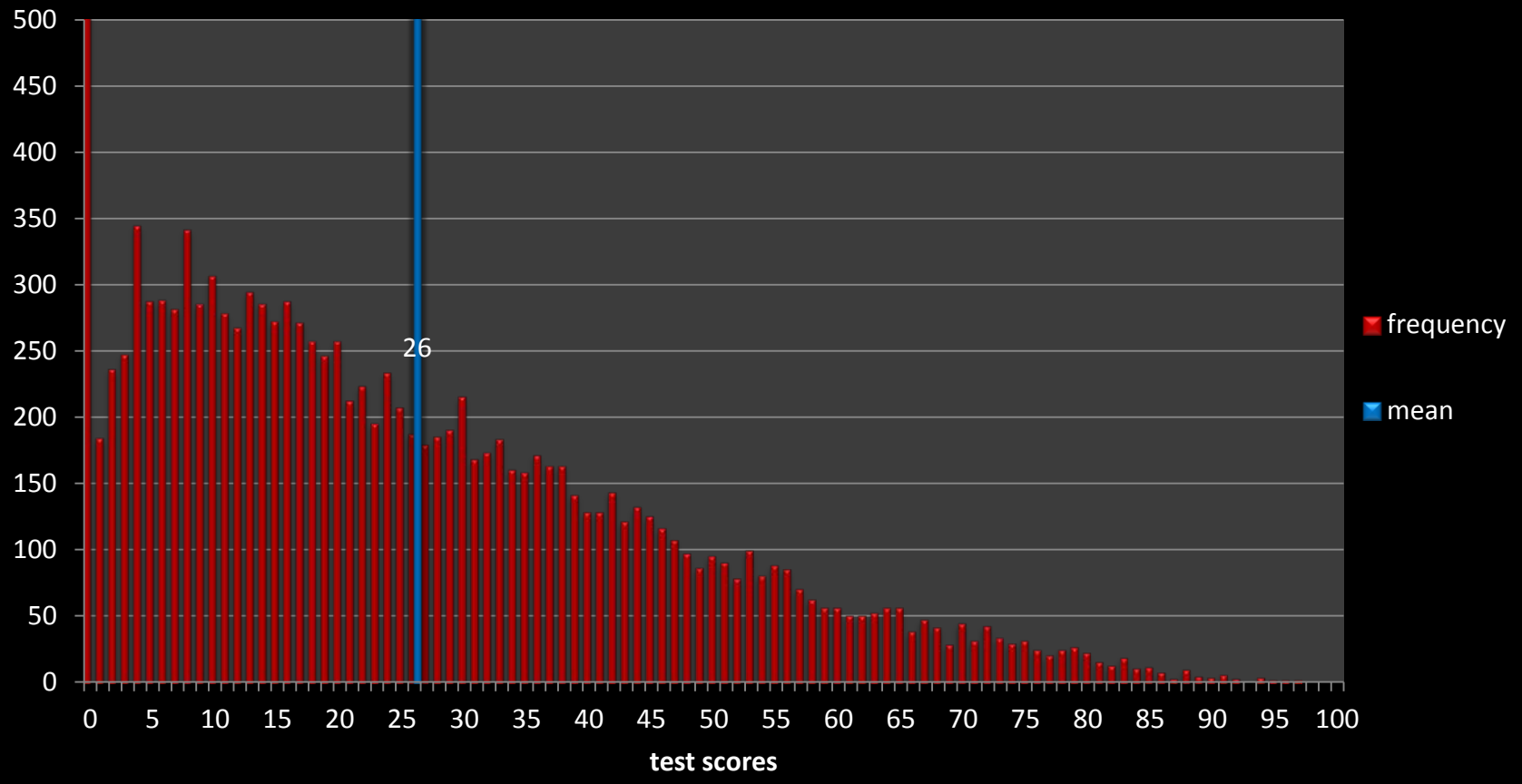
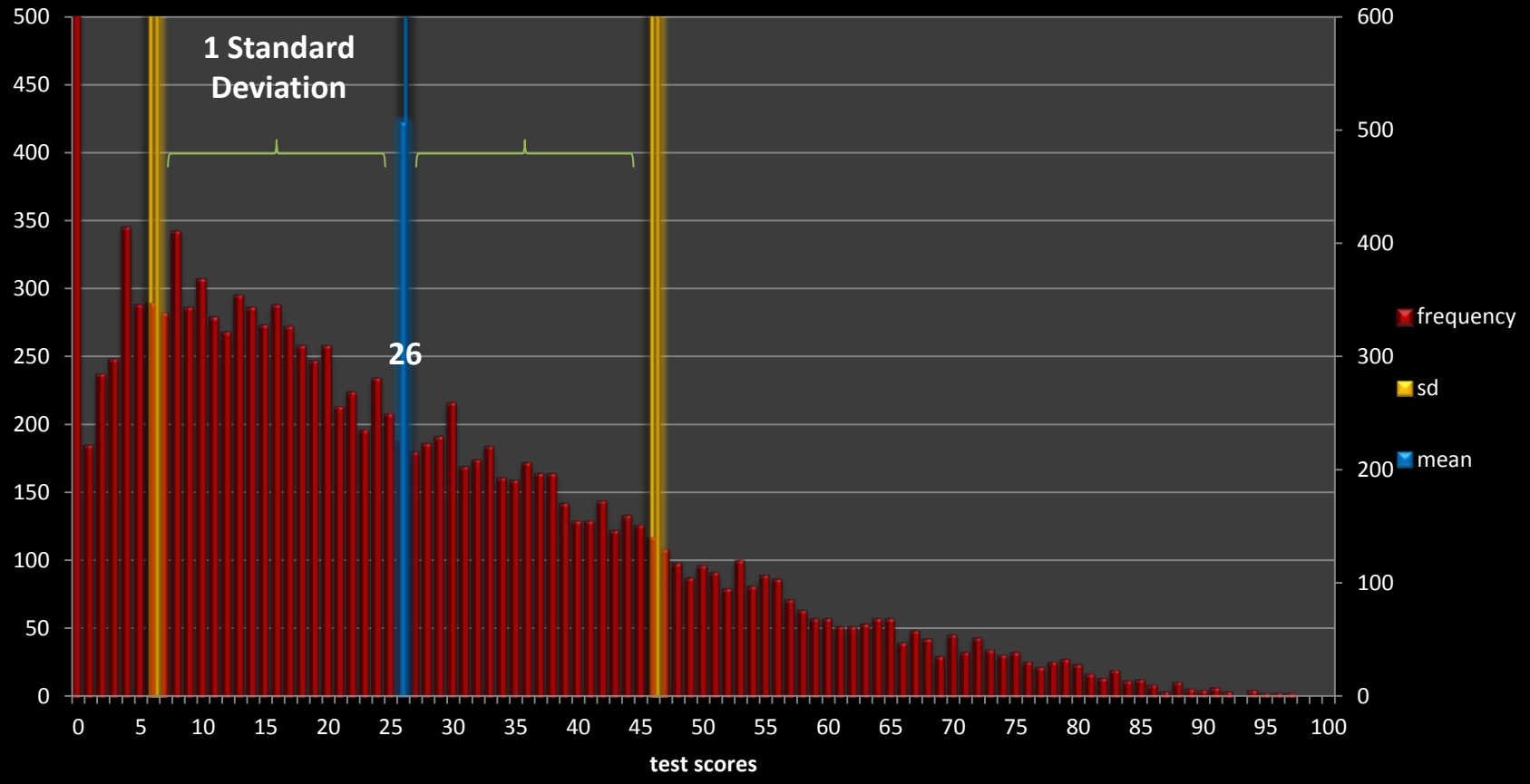
Baseline test scores



Mean = 26


500
450
400
350
300
250
200
150
100
50
0
26
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
test scores
frequency
mean

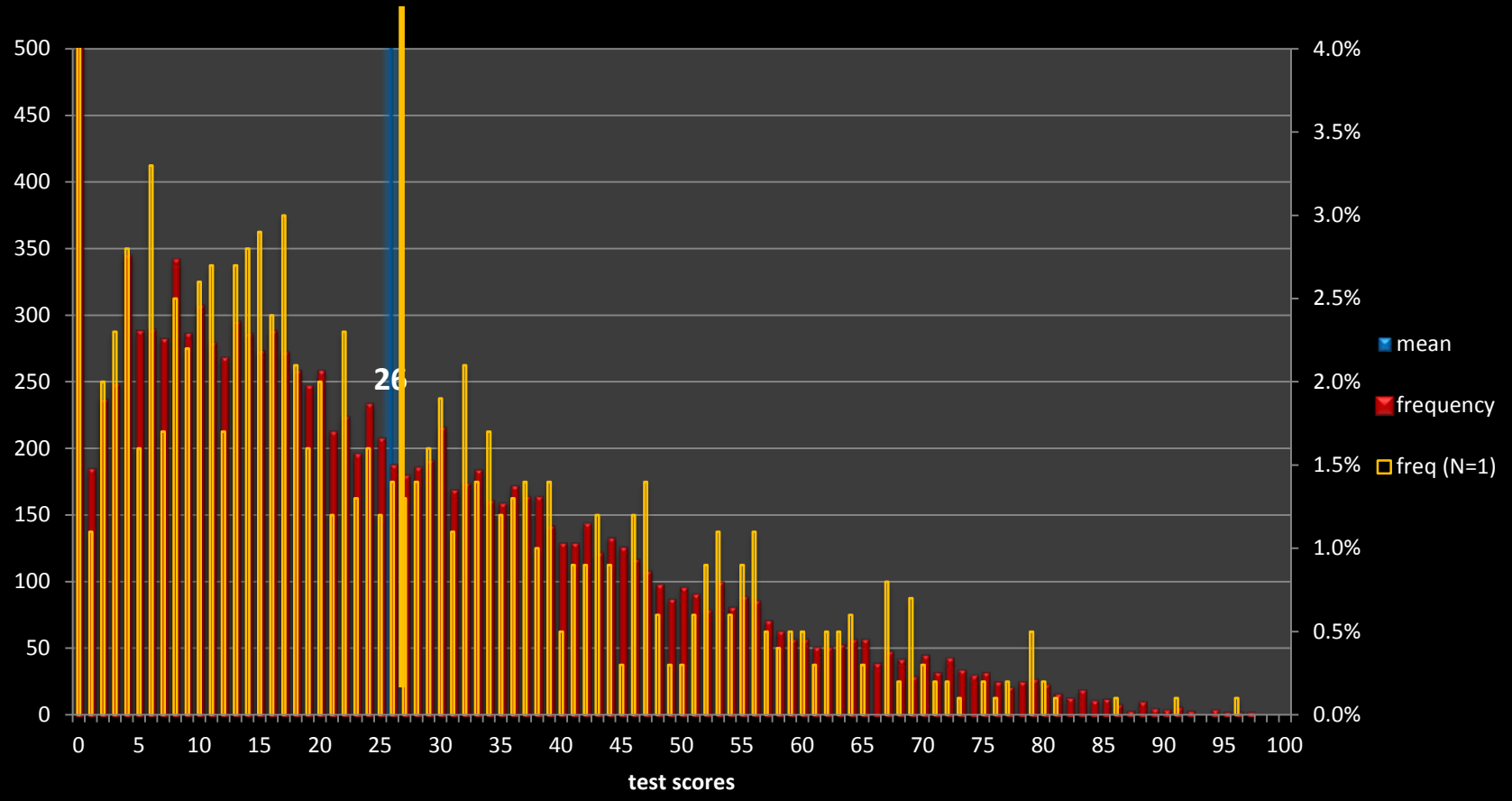
Standard Deviation = 20



Let's do an experiment

- Take 1 Random test score from the pile of 16,000 tests
- Write down the value
- Put the test back
- Do these three steps again
- And again
- 8,000 times
- This is like a random sample of 8,000 (*with replacement*)

What can we say about this sample?

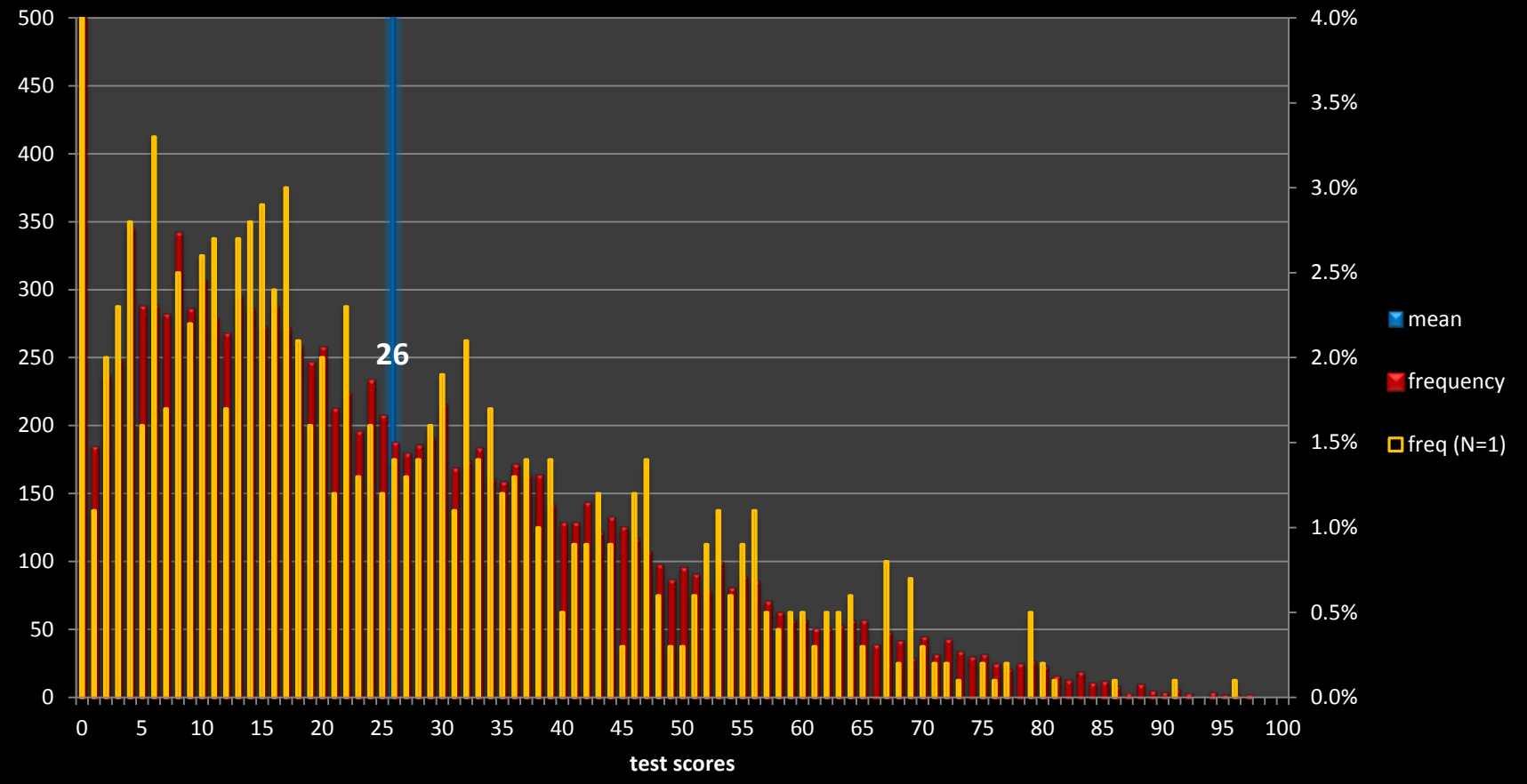


Good, the average of the sample is about 26…

But…

- … I remember that as my sample goes, up, isn't the sampling distribution supposed to turn into a bell curve?
- (Central Limit Theorem)
- Is it that my sample isn't large enough?

Population v. sampling distribution



This is the distribution of my sample of 8,000 students!

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- Sampling distributions
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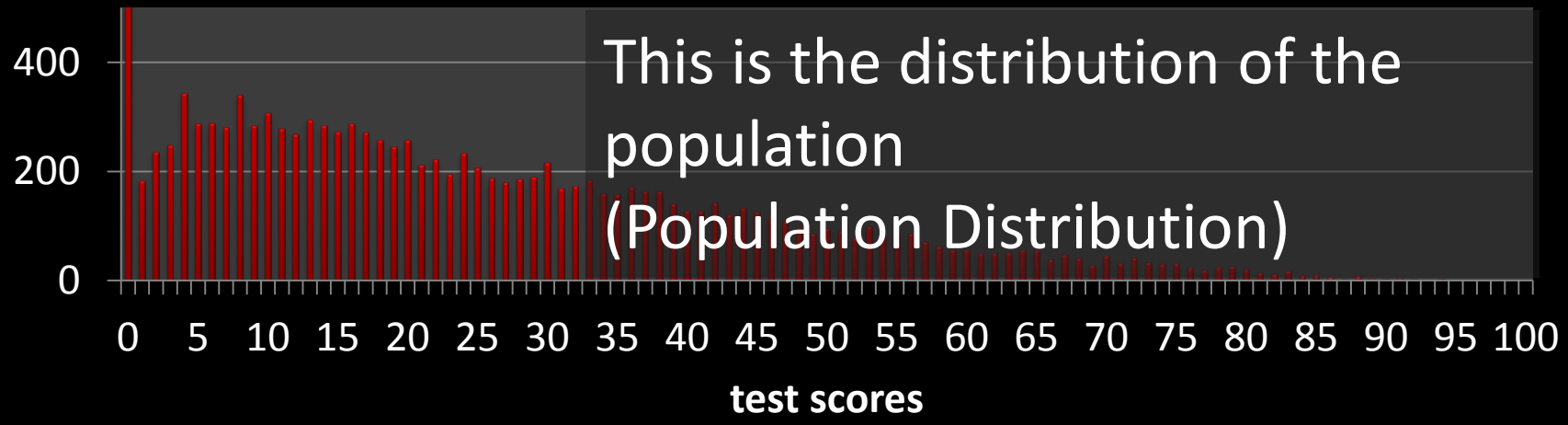
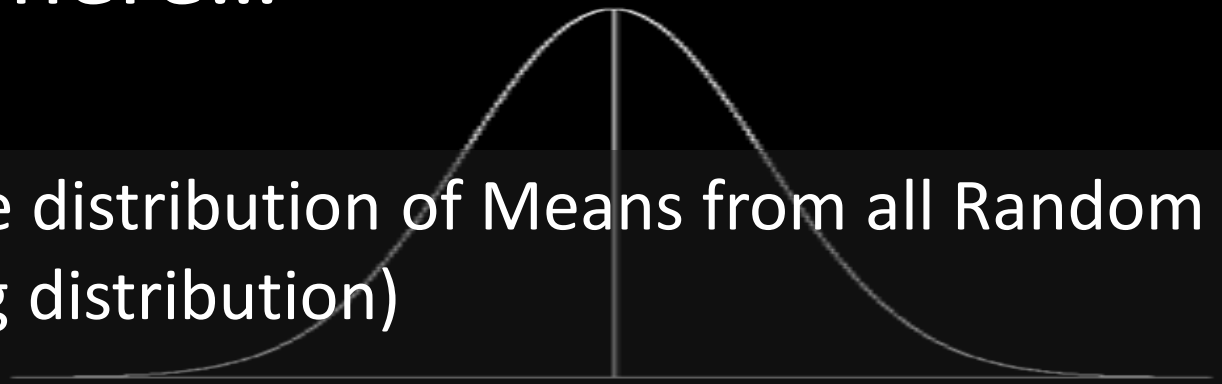
How do we get from here…

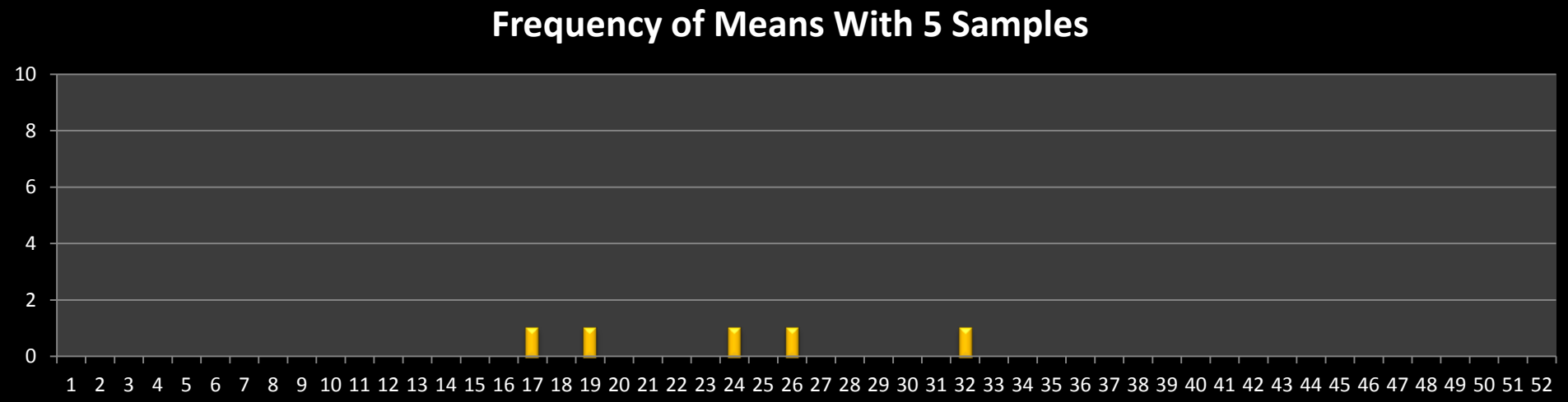


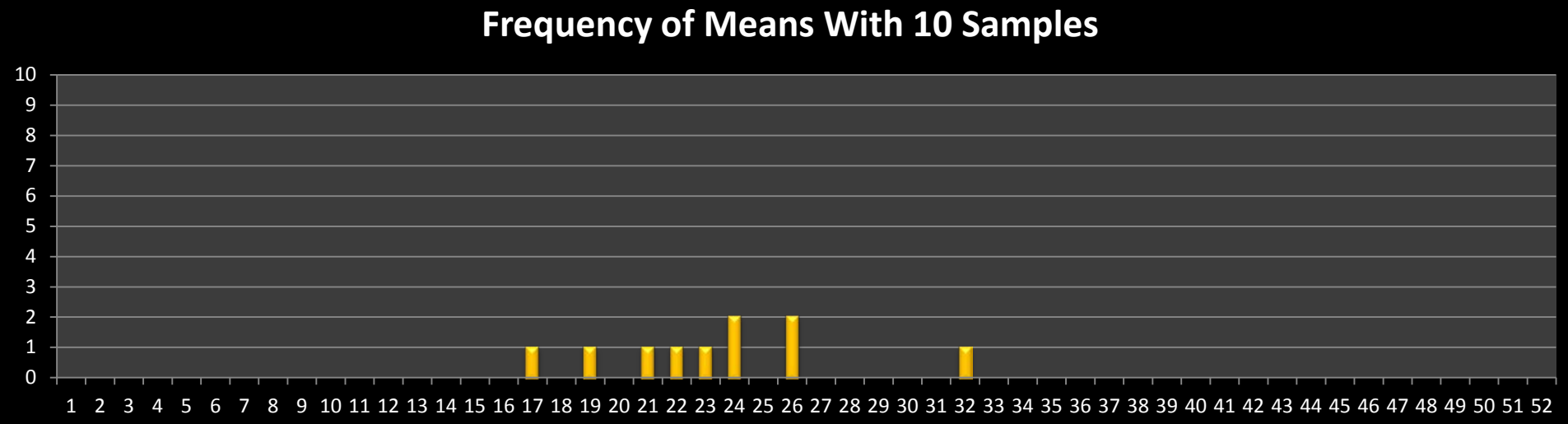
To here…



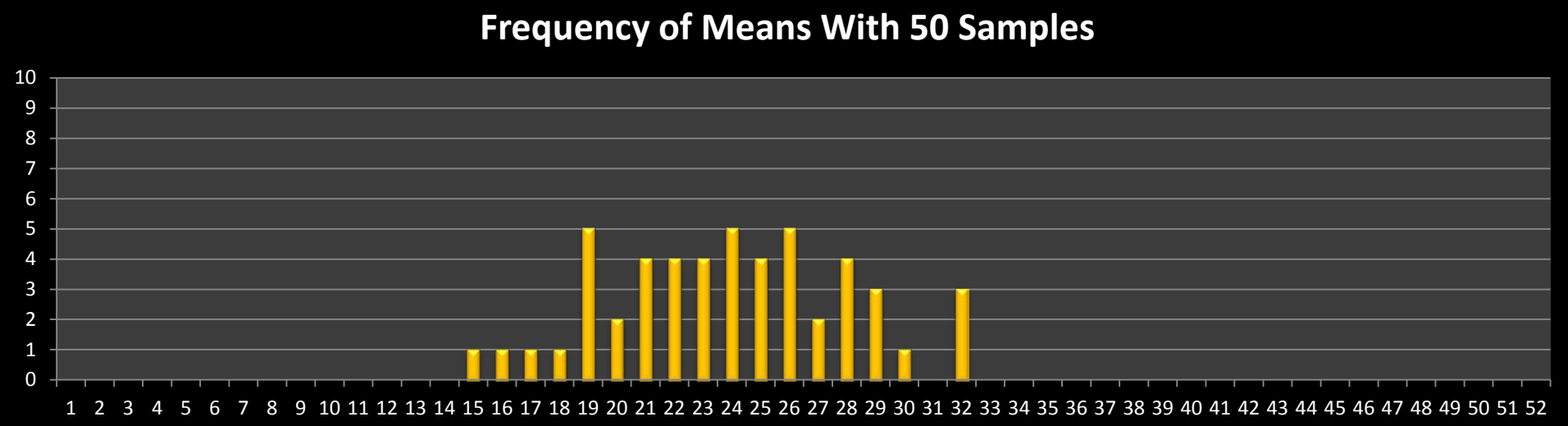
This is the distribution of Means from all Random Samples (Sampling distribution)

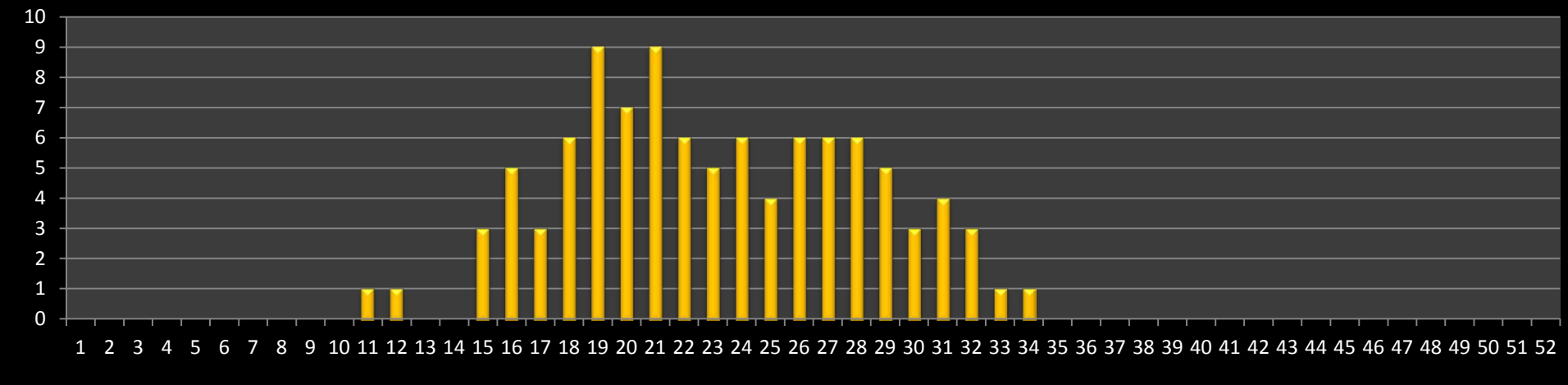
Draw 10 random students, take the average, plot it: Do this 5 & 10 times.



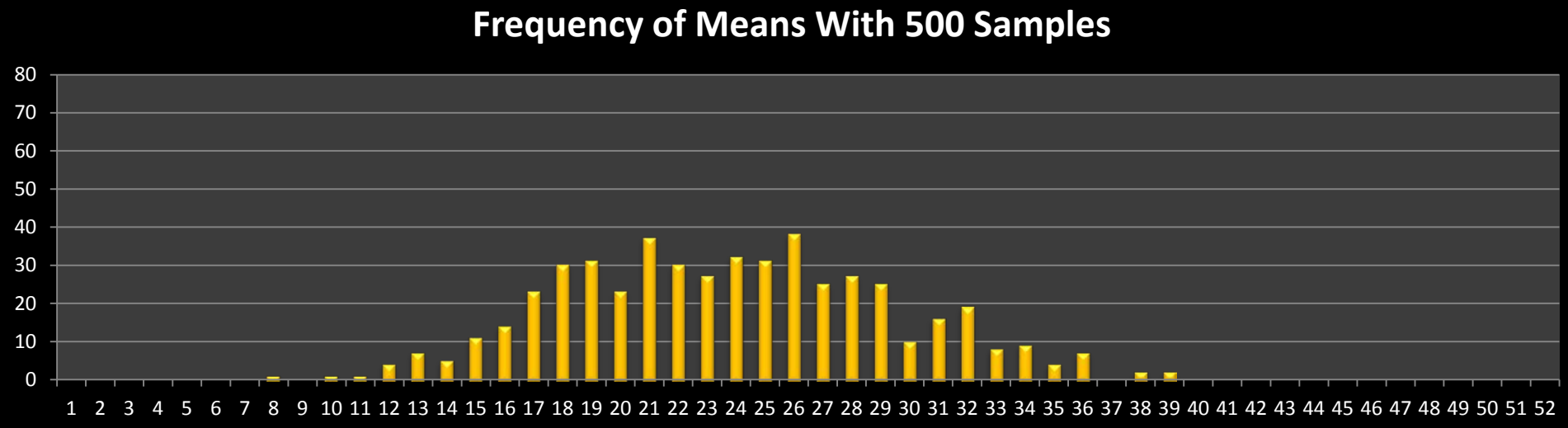


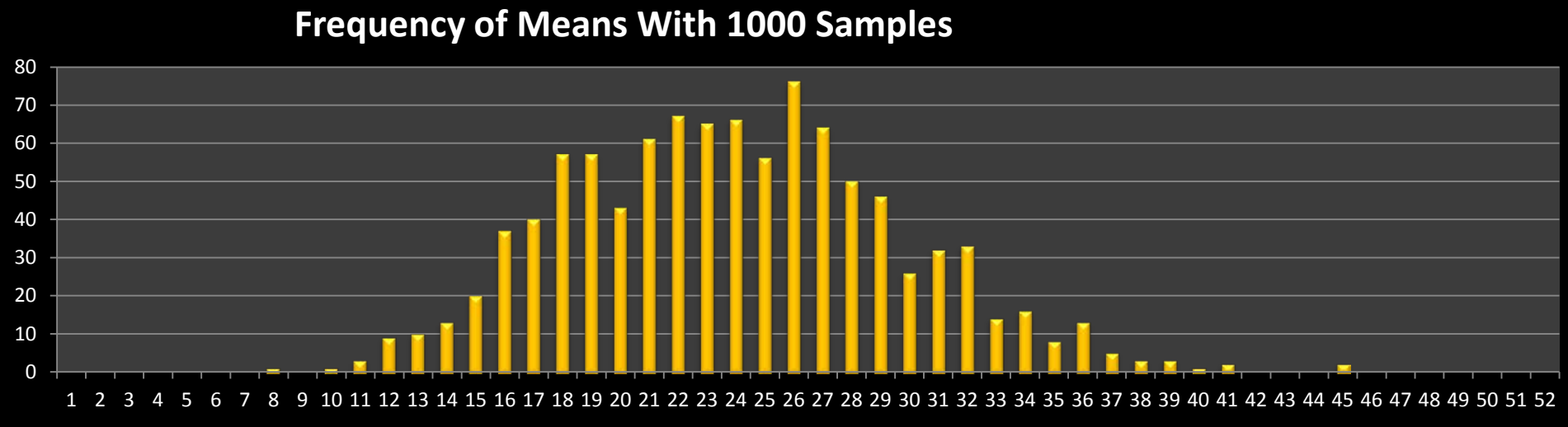
Draw 10 random students: 50 and 100 times





Draws 10 random students: 500 and 1000 times





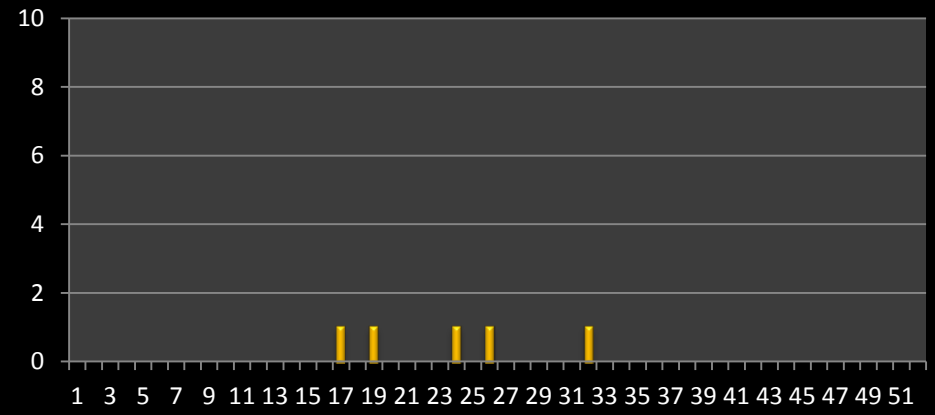
Draw 10 Random students

- This is like a sample size of 10
- What happens if we take a sample size of 50?

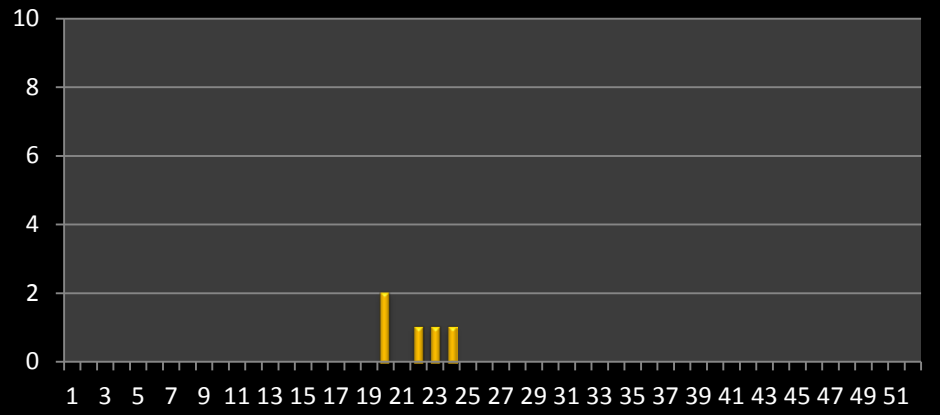
N = 10

N = 50

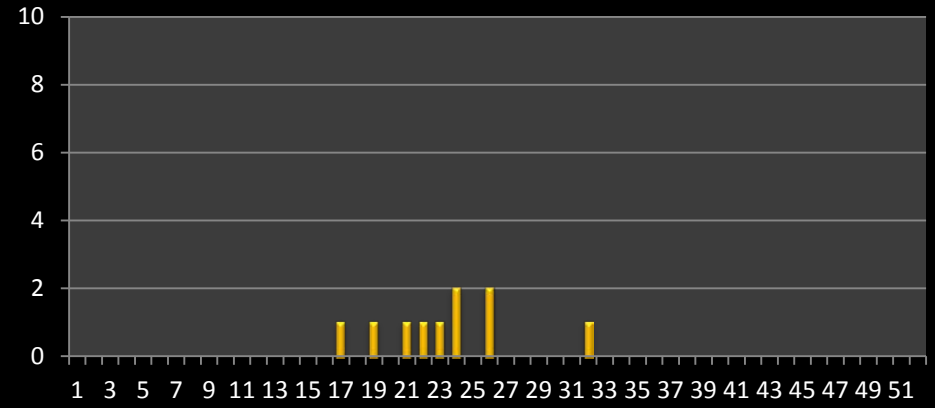
Frequency of Means With 5 Samples



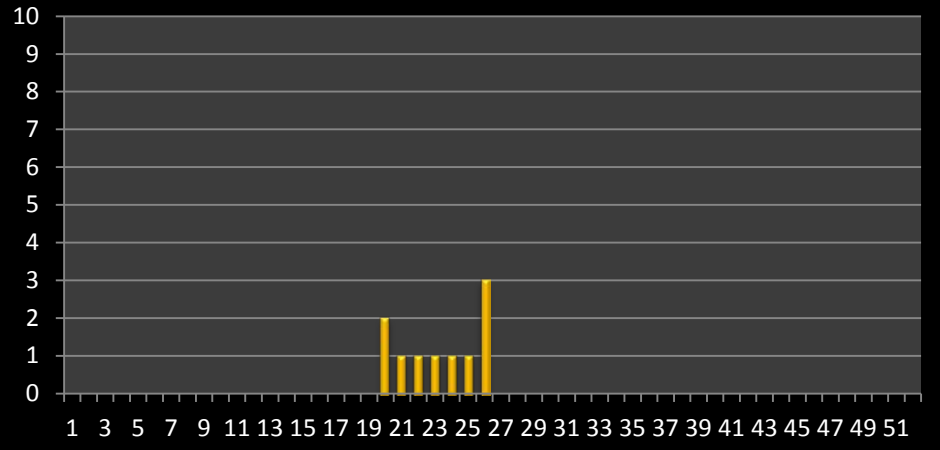
Frequency of Means With 5 Samples



Frequency of Means With 10 Samples

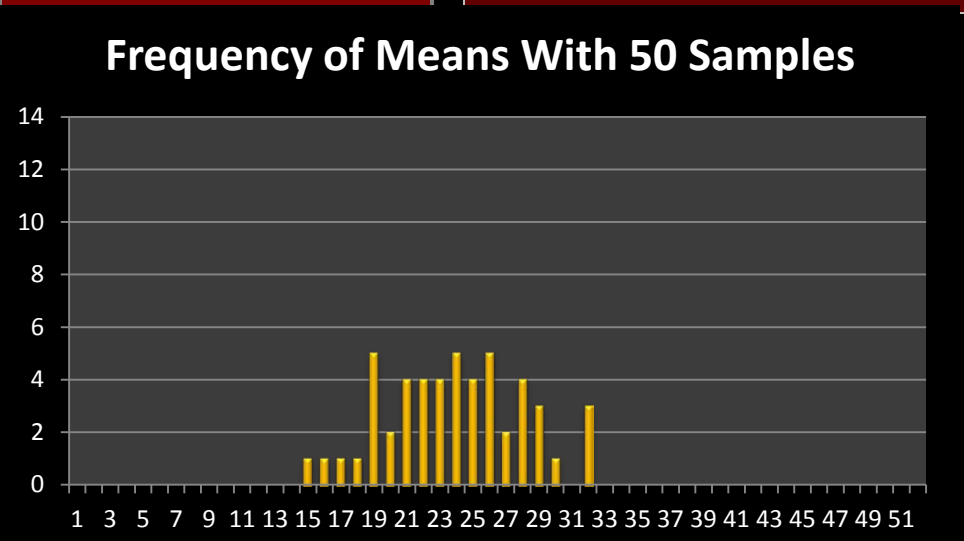


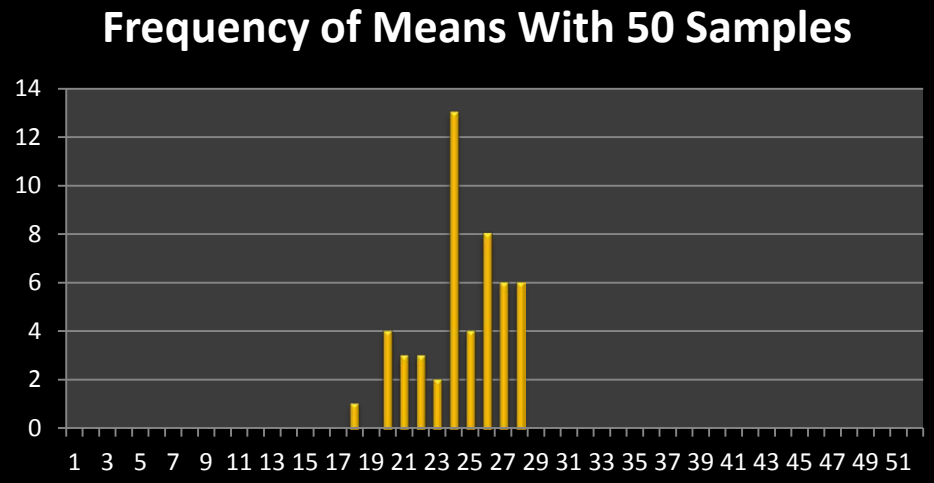
Frequency of Means With 10 Samples

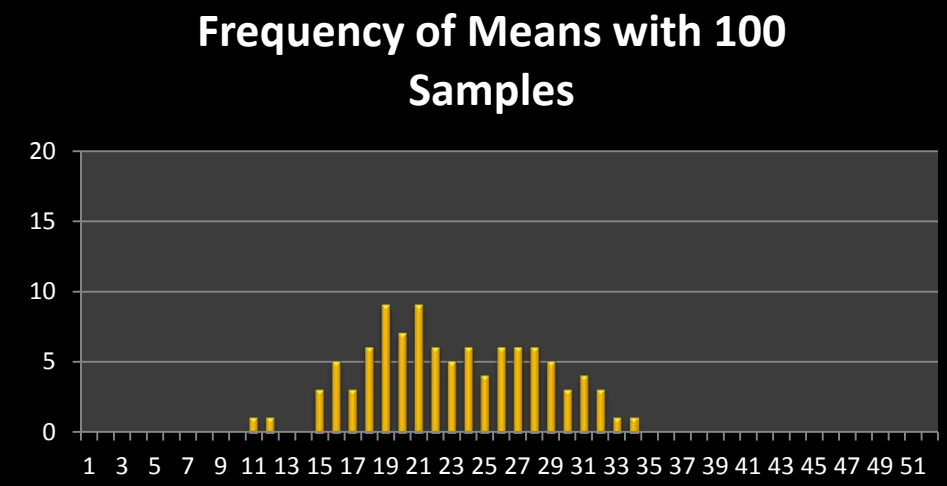


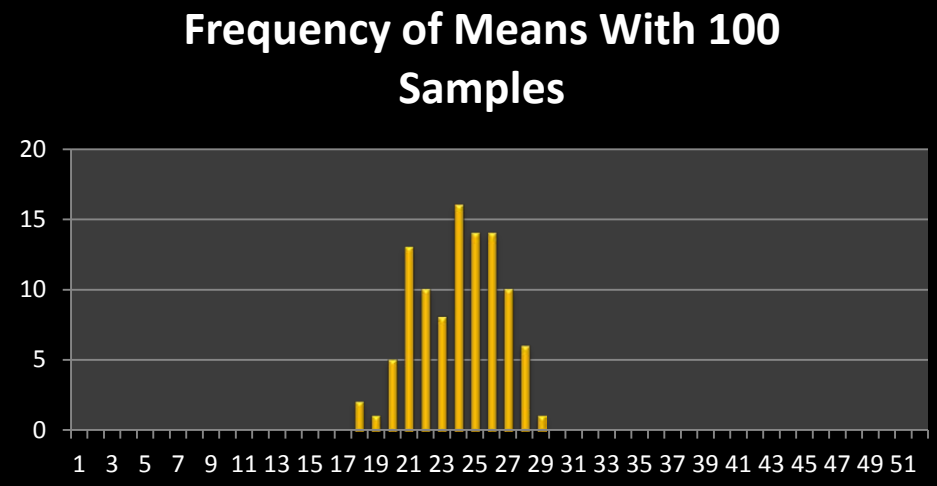
Draws of 10

Draws of 50





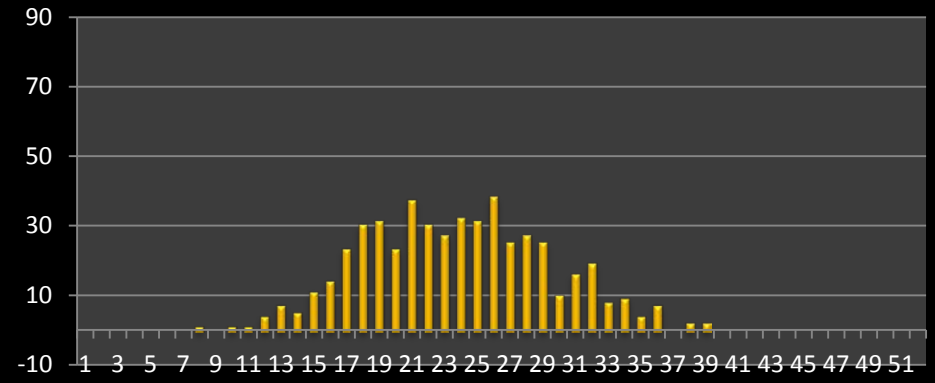




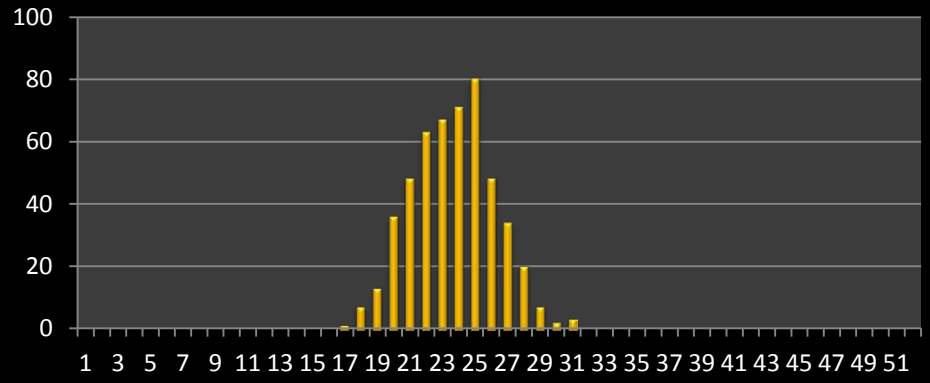
Draws of 10

Draws of 50

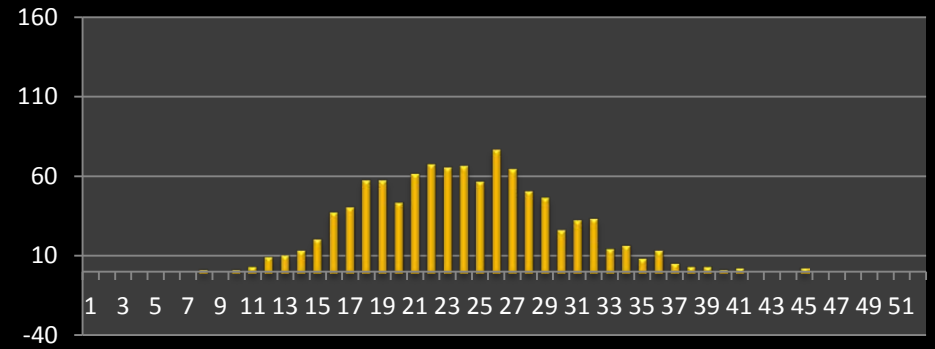
Frequency of Means With 500 Samples



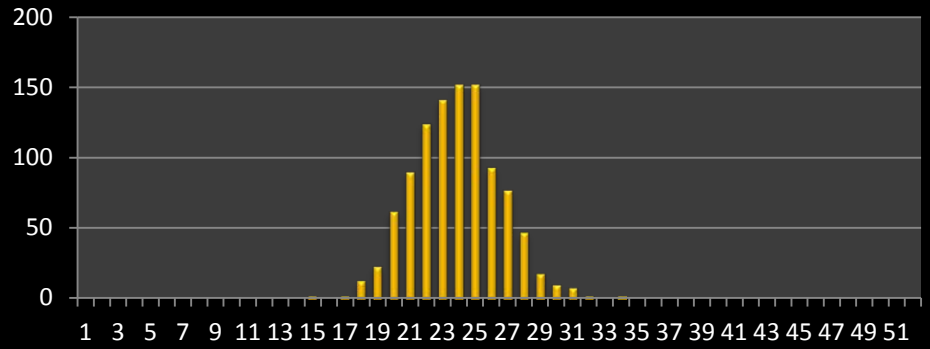
Frequency of Means With 500 Samples



Frequency of Means With 1000 Samples



Frequency of Means With 1000 Samples



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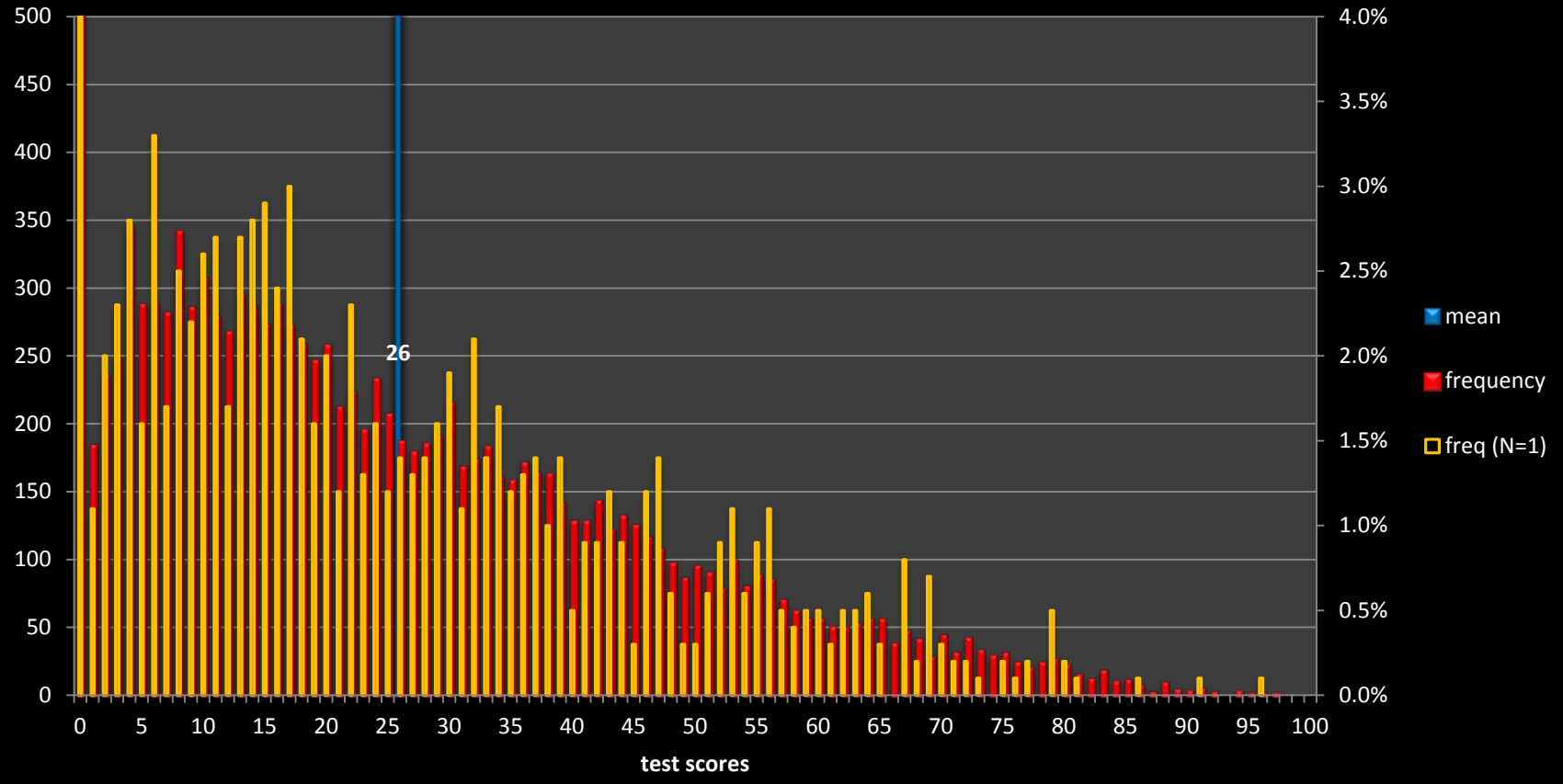
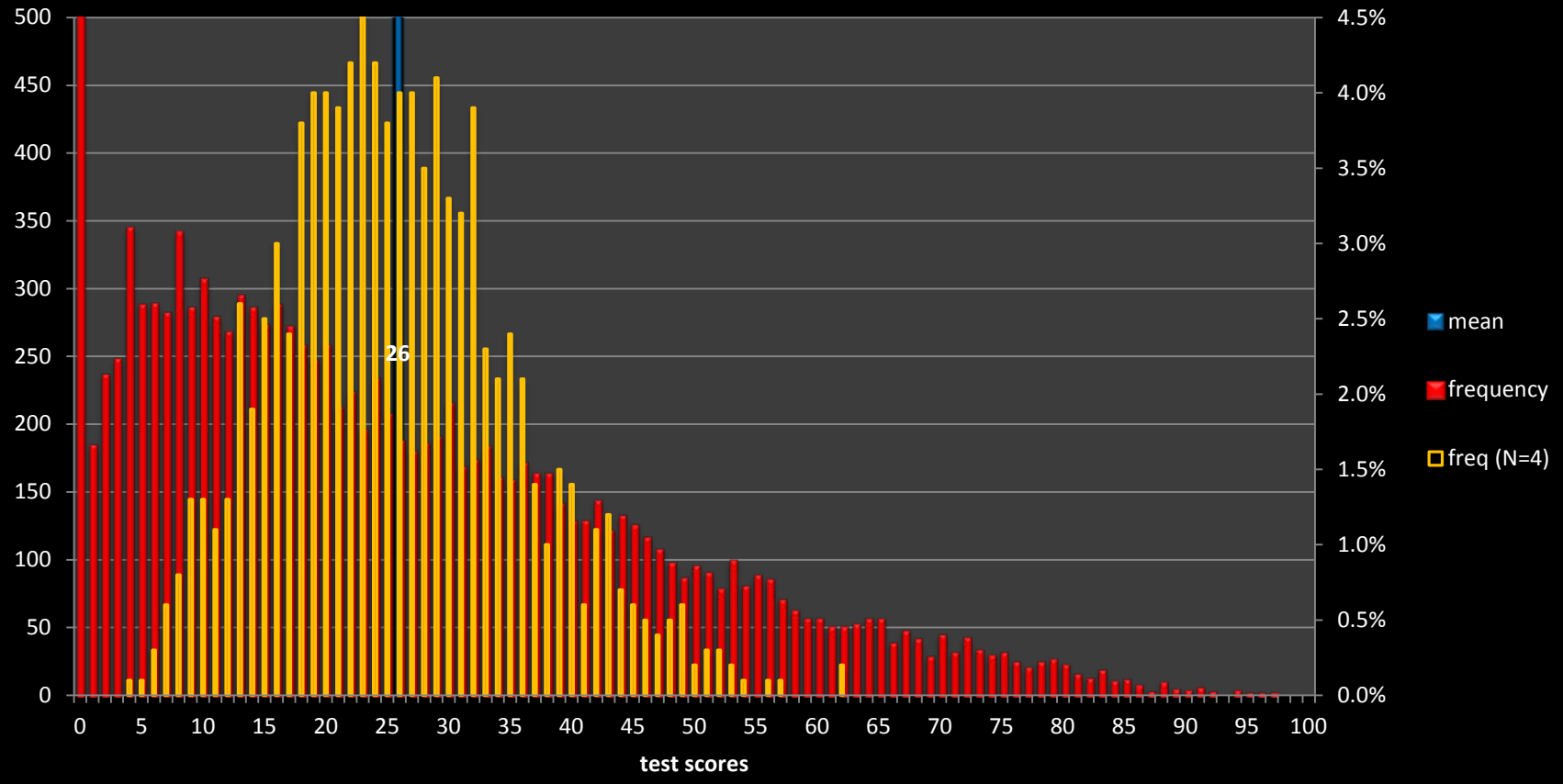
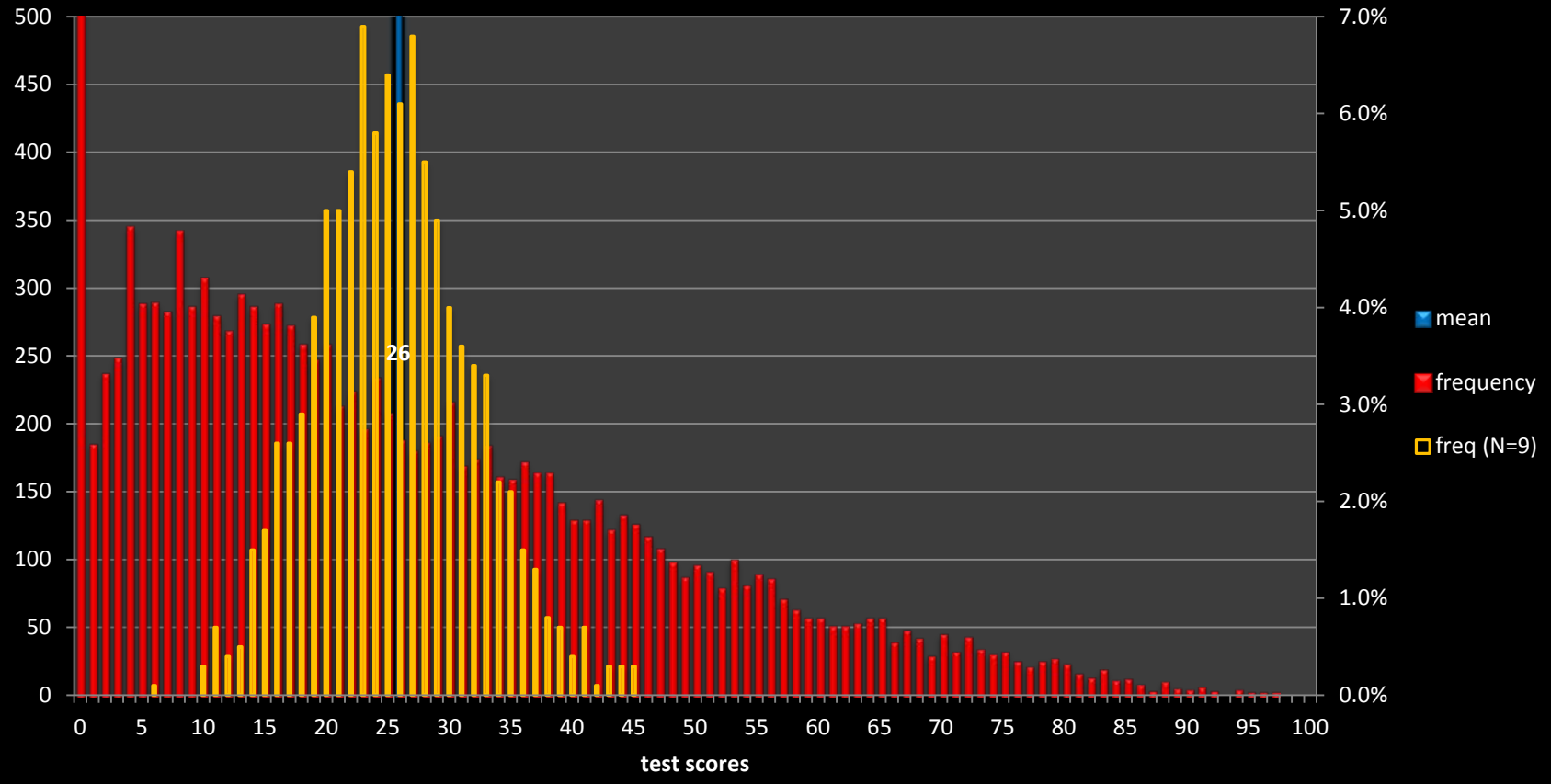
Population & sampling distribution: Draw 1 random student (from 8,000)



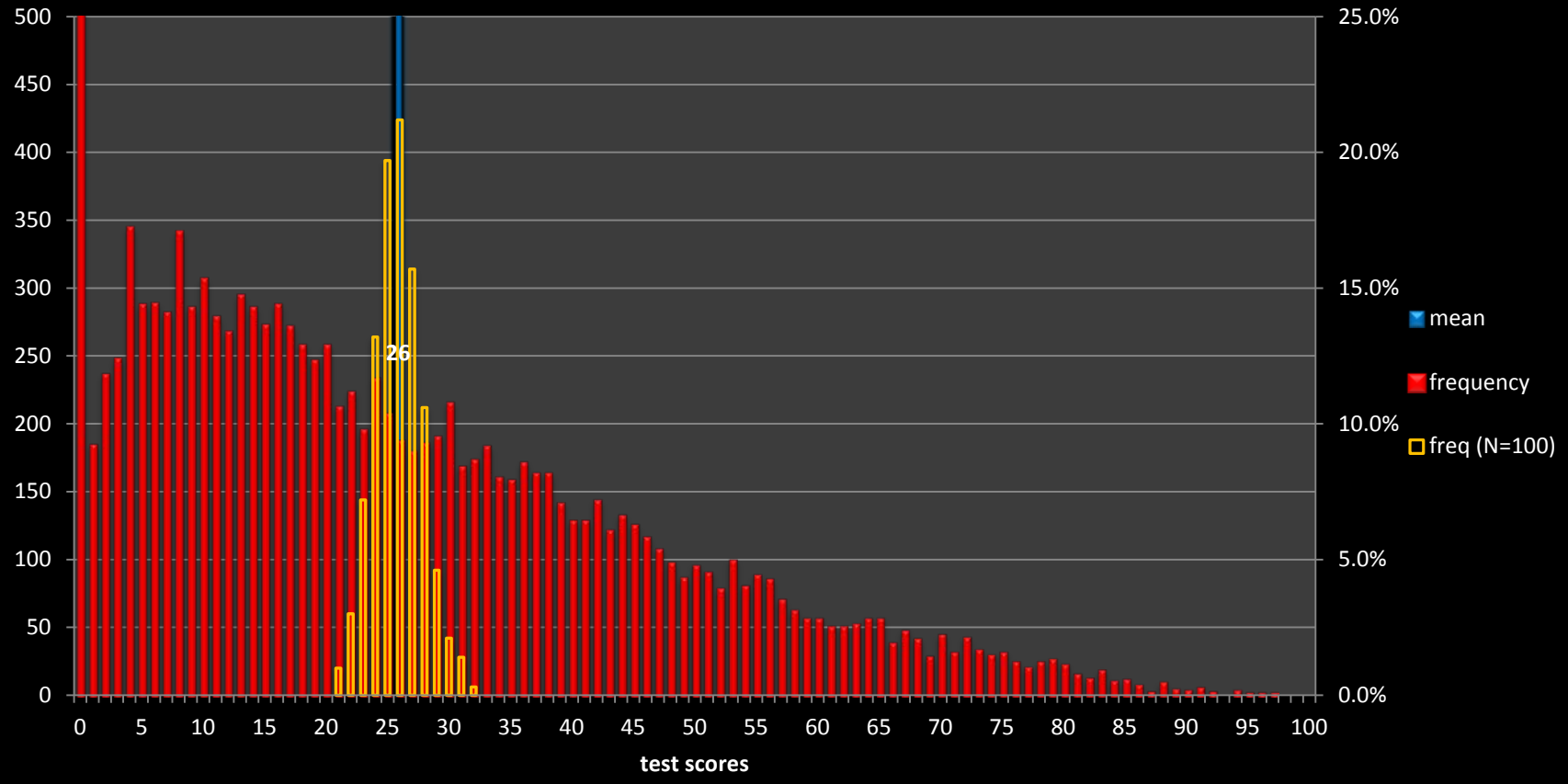
Sampling Distribution: Draw 4 random students (N=4)



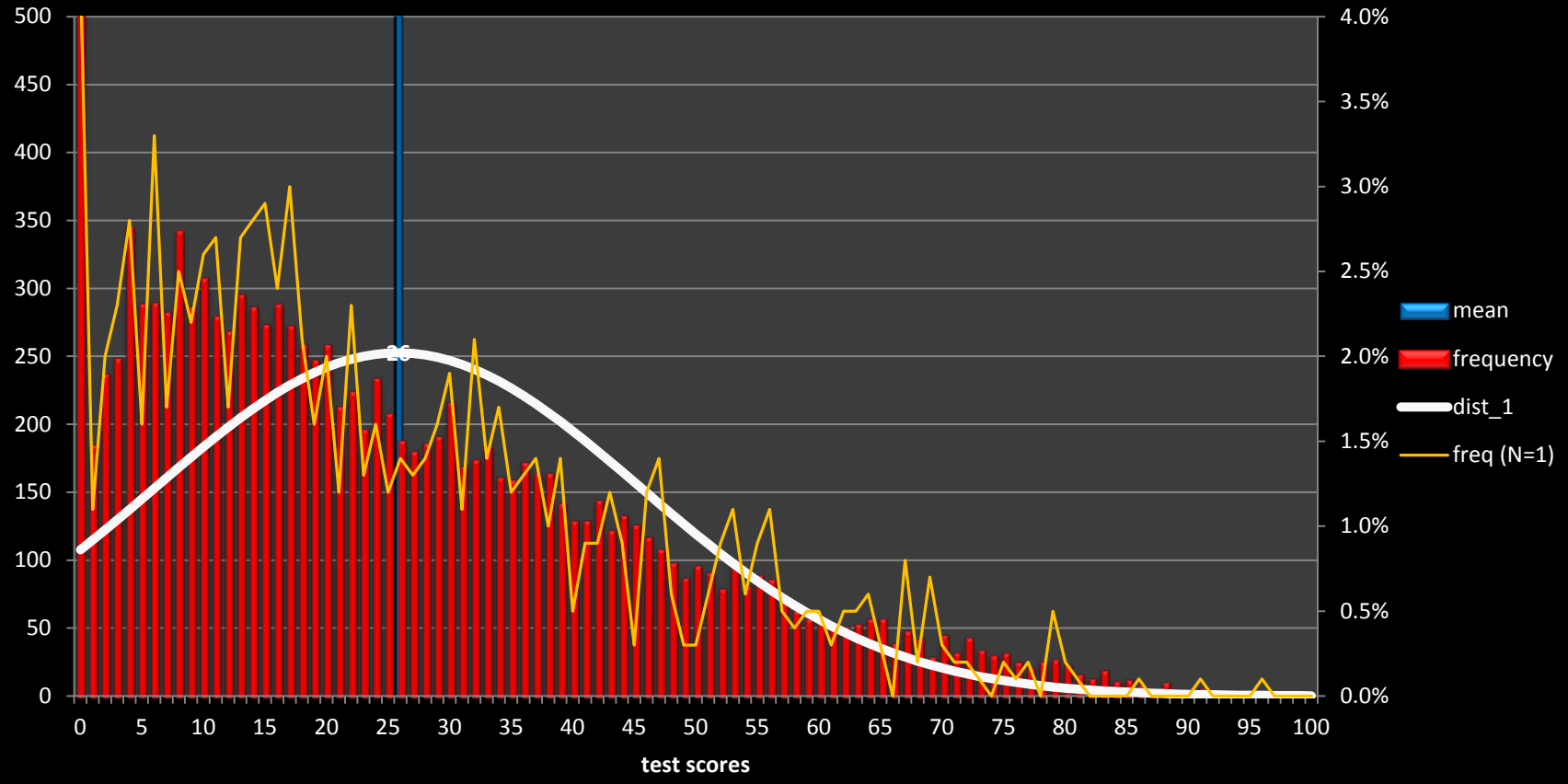
Law of Large Numbers : N=9



Law of Large Numbers: N =100

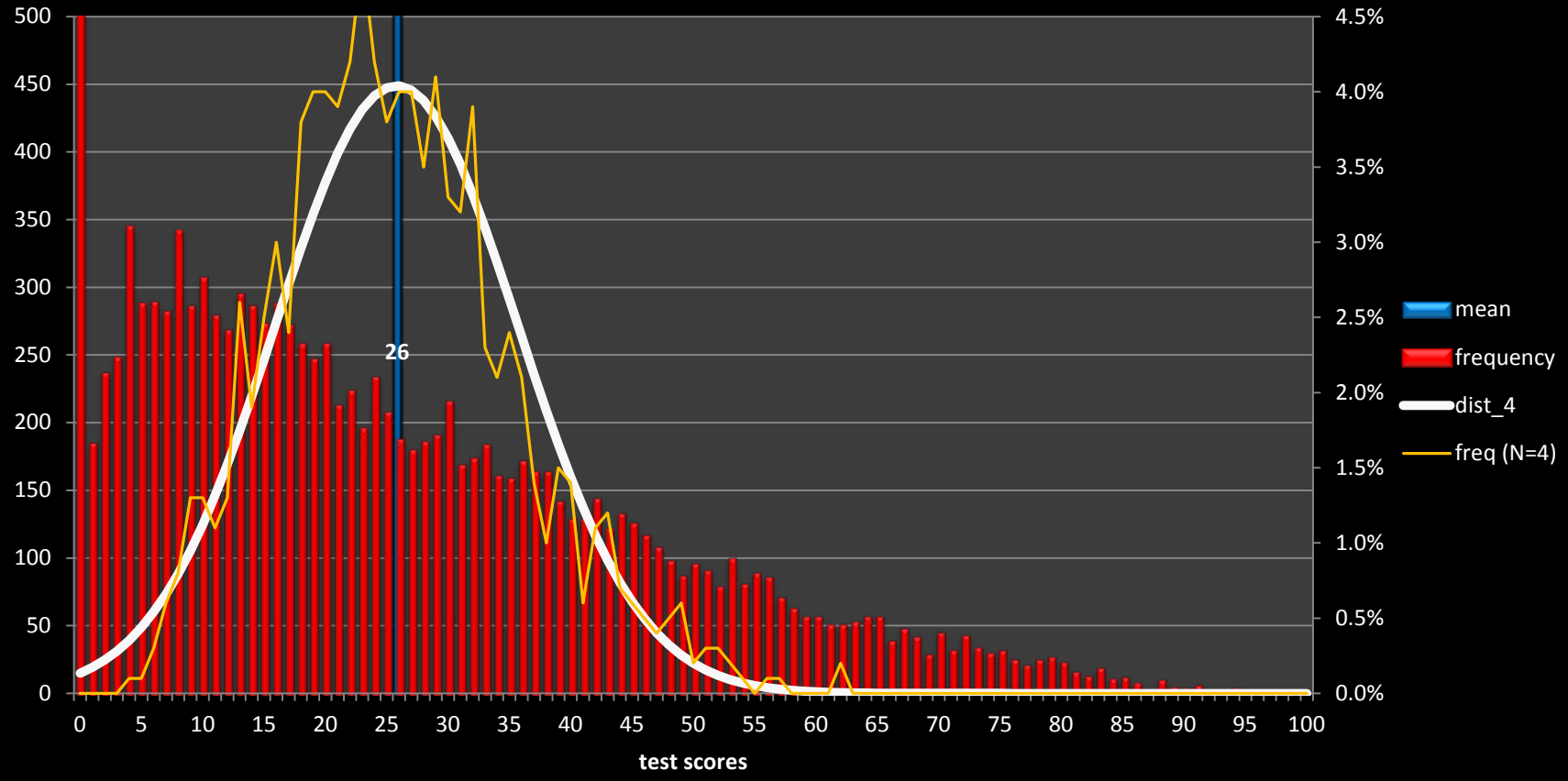


Central Limit Theorem: N=1

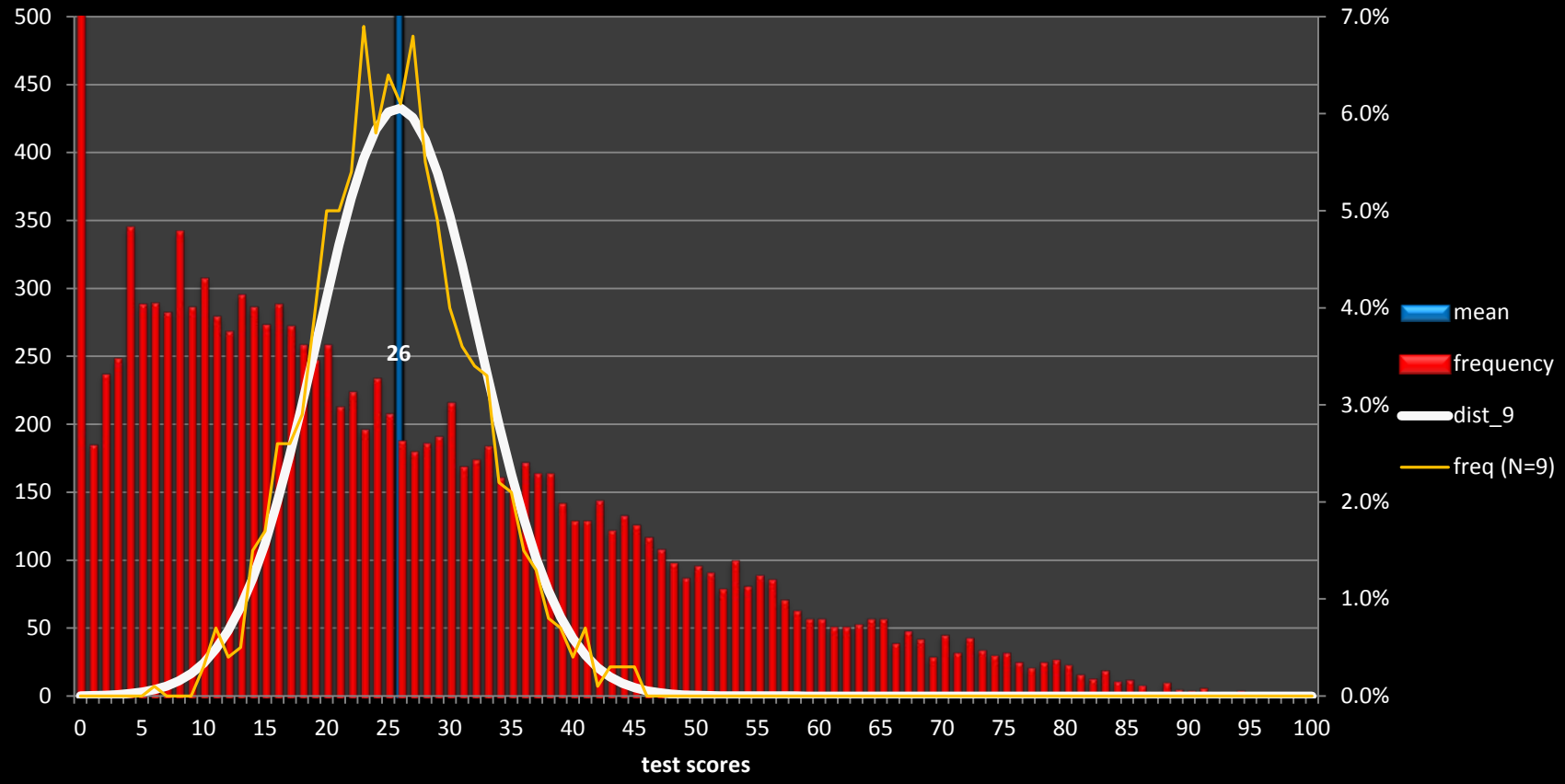


- The white line is a theoretical distribution

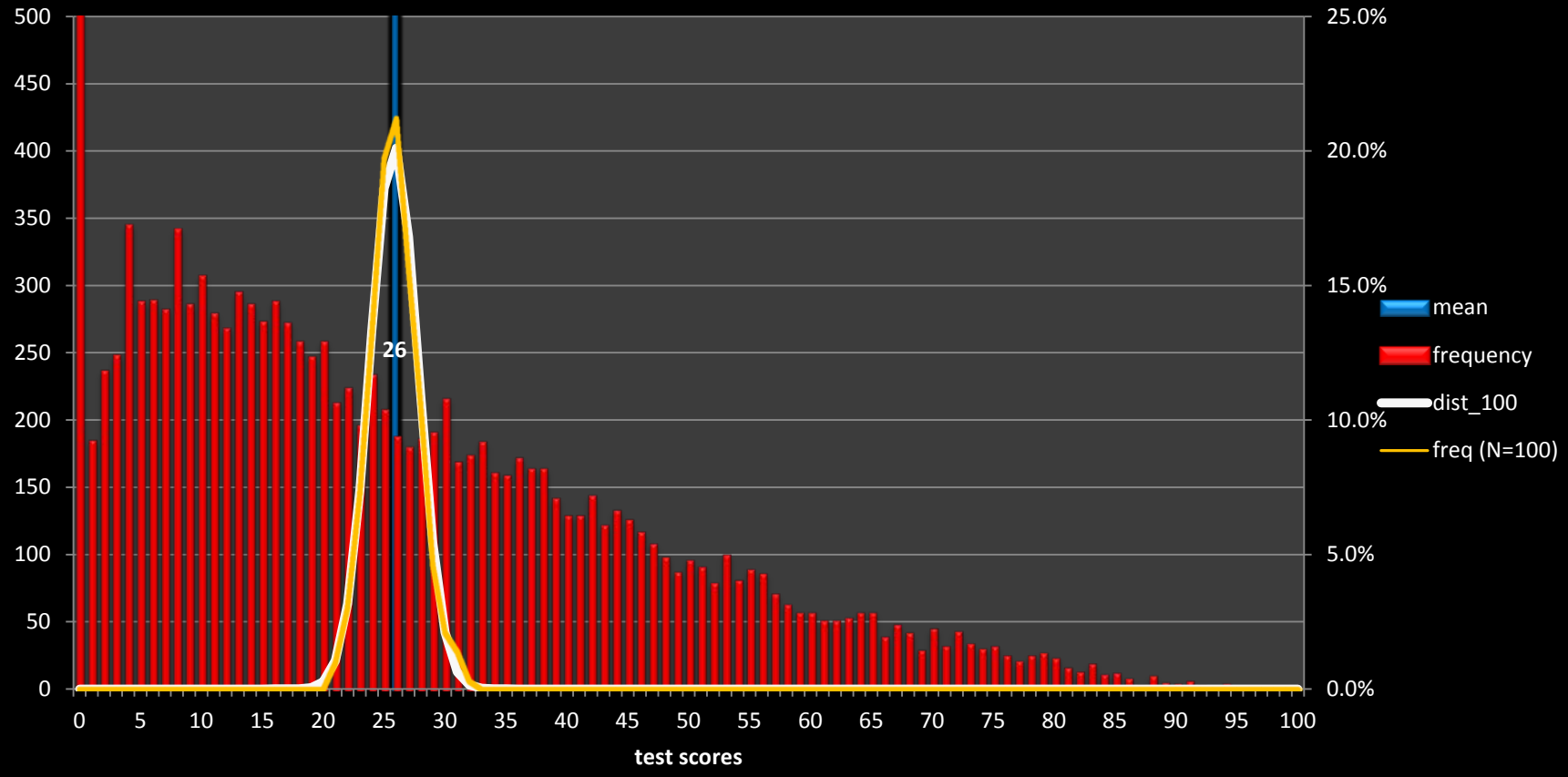
Central Limit Theorem : N=4



Central Limit Theorem : N=9



Central Limit Theorem : N =100



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Standard deviation/error

- What's the difference between the standard deviation and the standard error?
- The standard error = the standard deviation of the sampling distributions

Variance and Standard Deviation

- Variance = 400

$$\sigma^2 = \frac{\sum(Observation\ Value\ - Average)^2}{N}$$

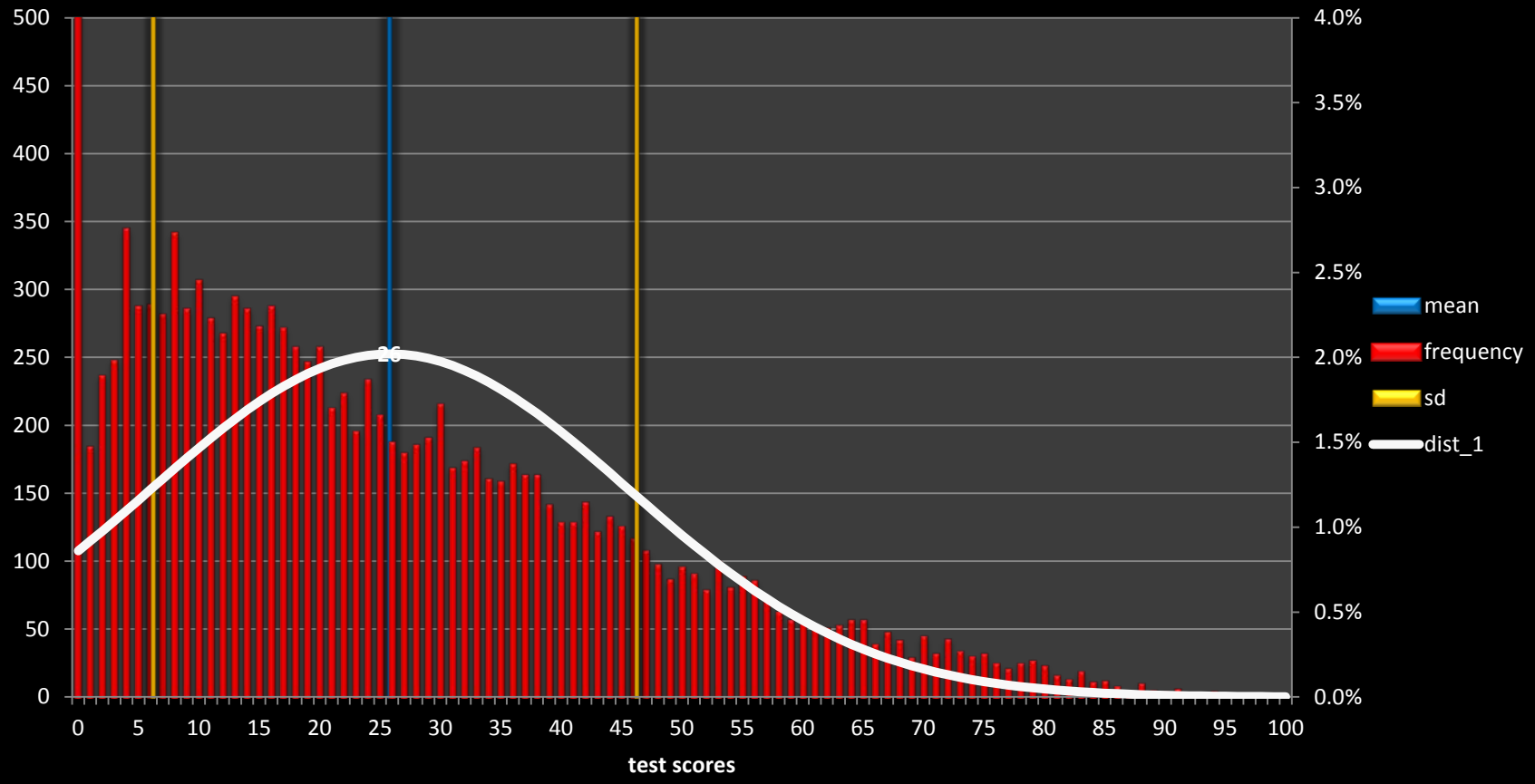
- Standard Deviation = 20

$$\sigma = \sqrt{Variance}$$

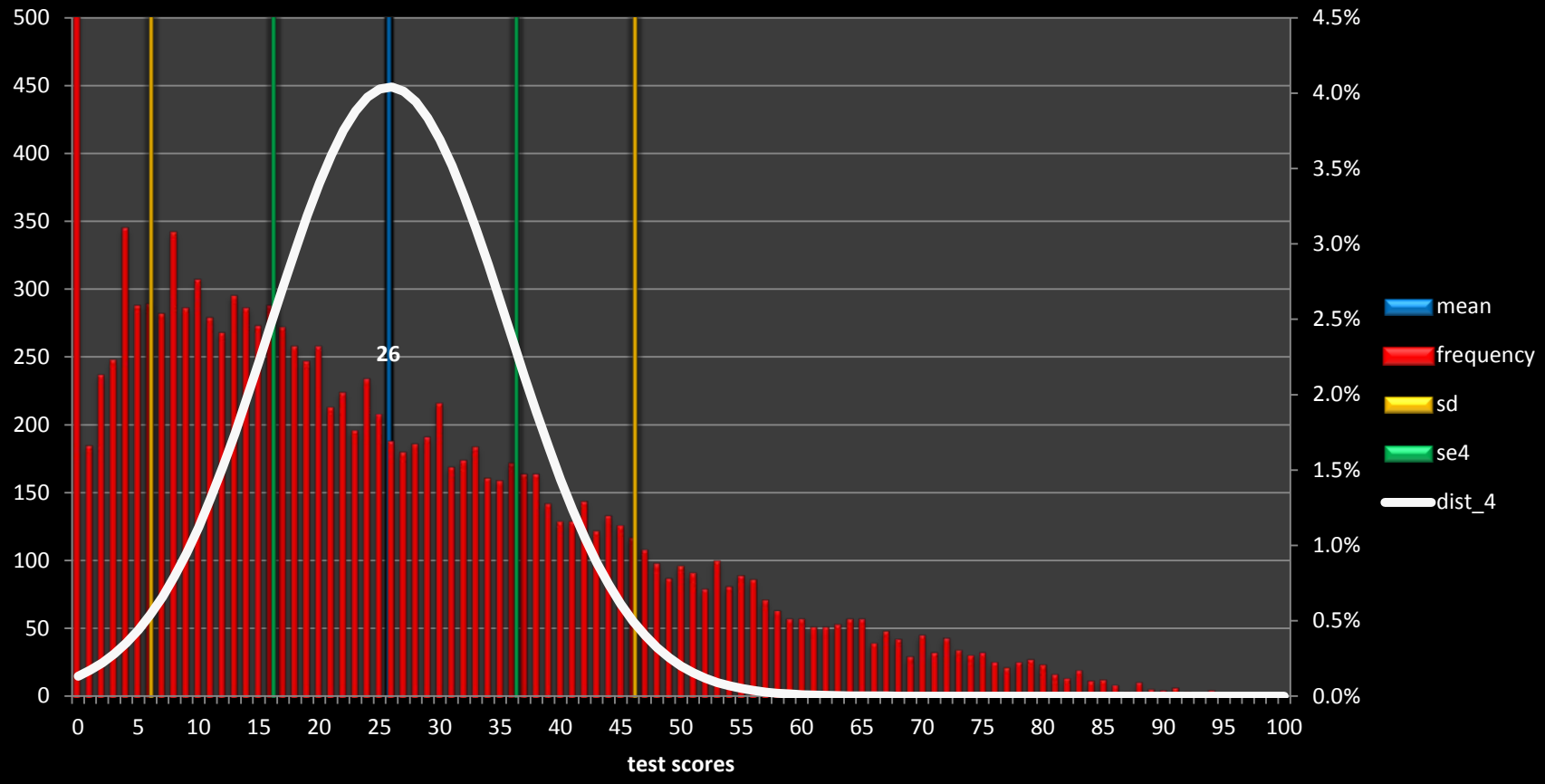
- Standard Error = $^{20}/_{\sqrt{N}}$

$$SE = {}^{\sigma}/_{\sqrt{N}}$$

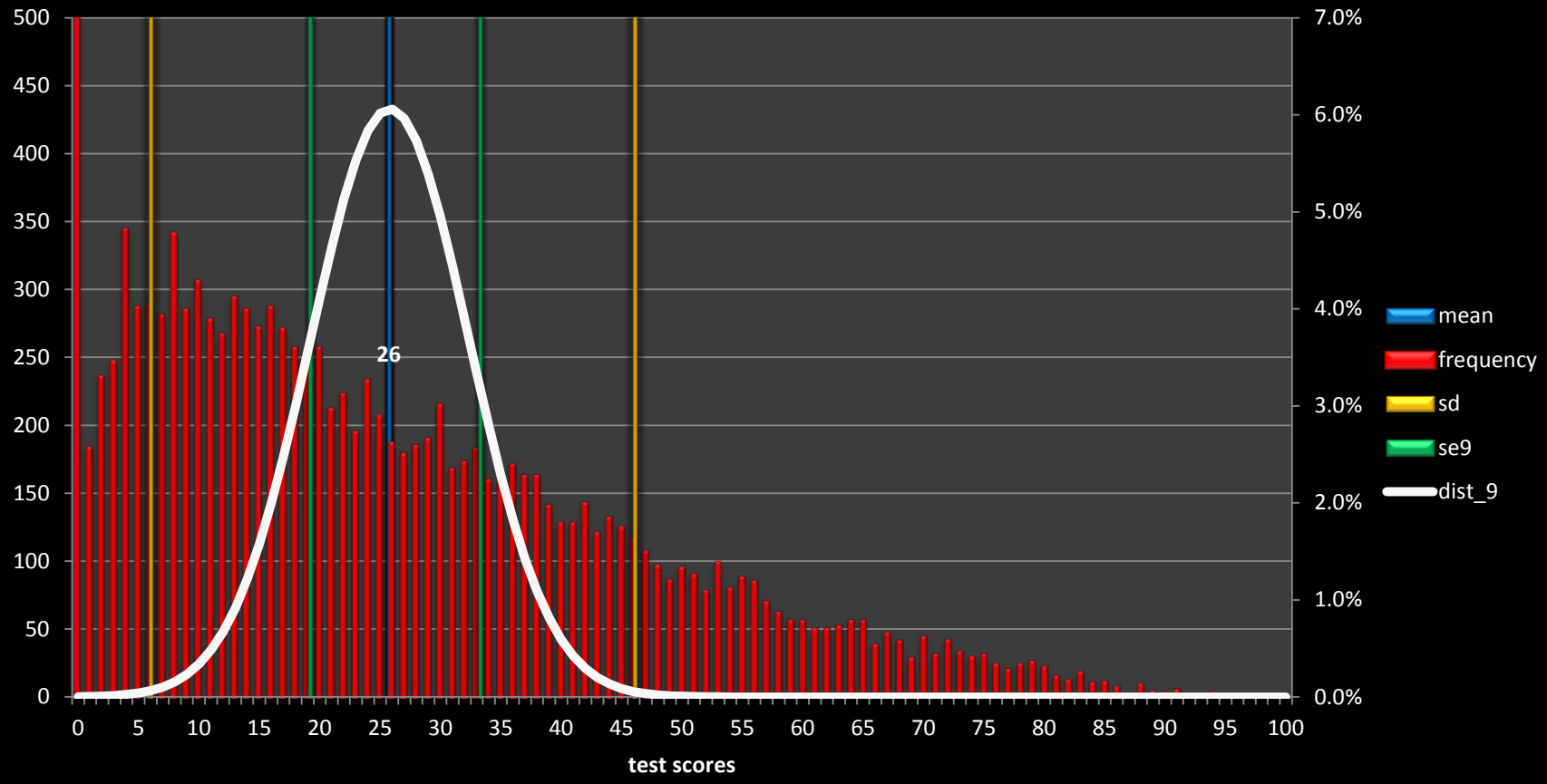
Standard Deviation/ Standard Error



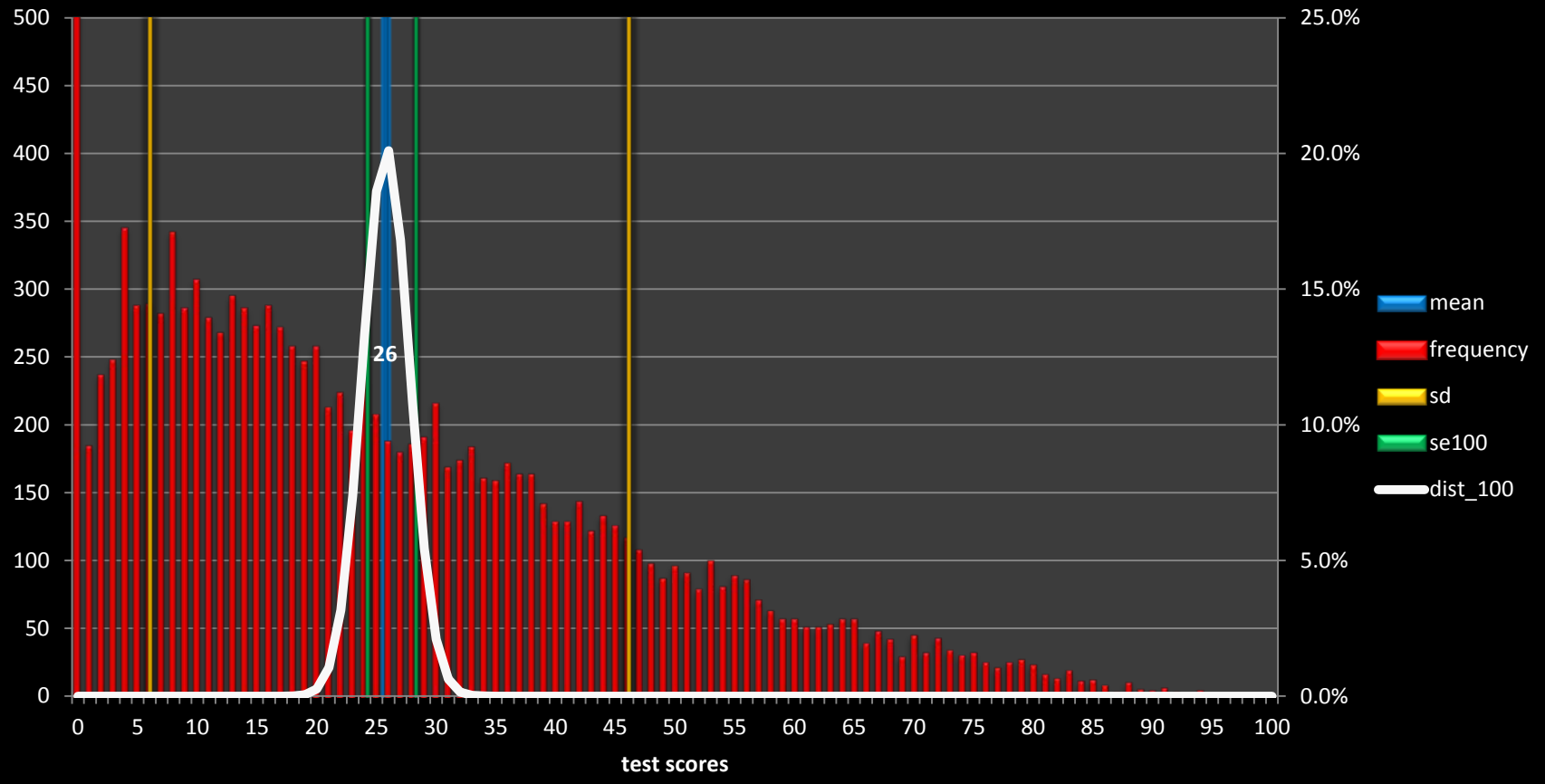
Sample size ↑ x4, SE ↓ ½



Sample size ↑ x9, SE ↓ ?



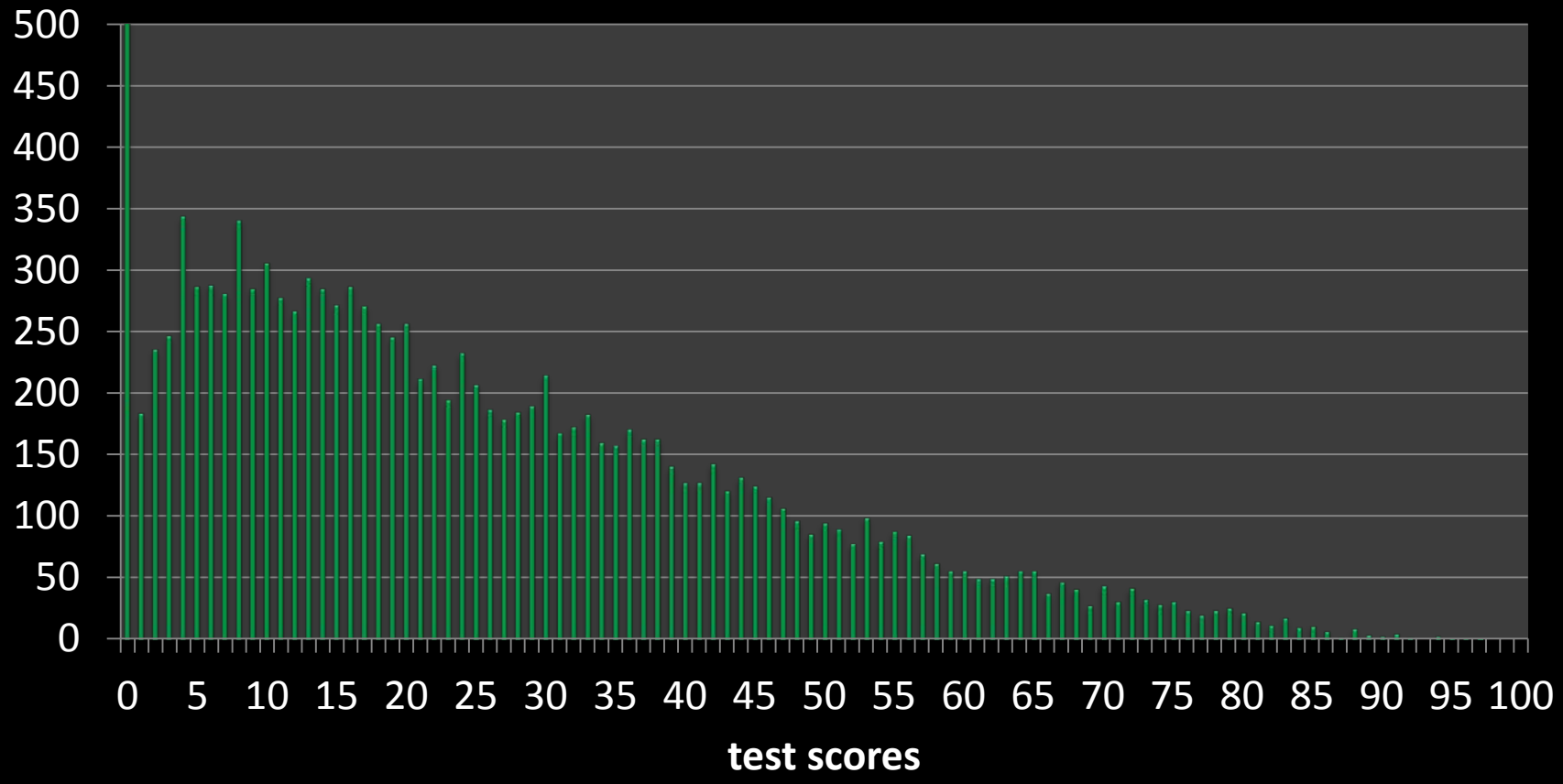
Sample size ↑ x100, SE ↓?



Outline

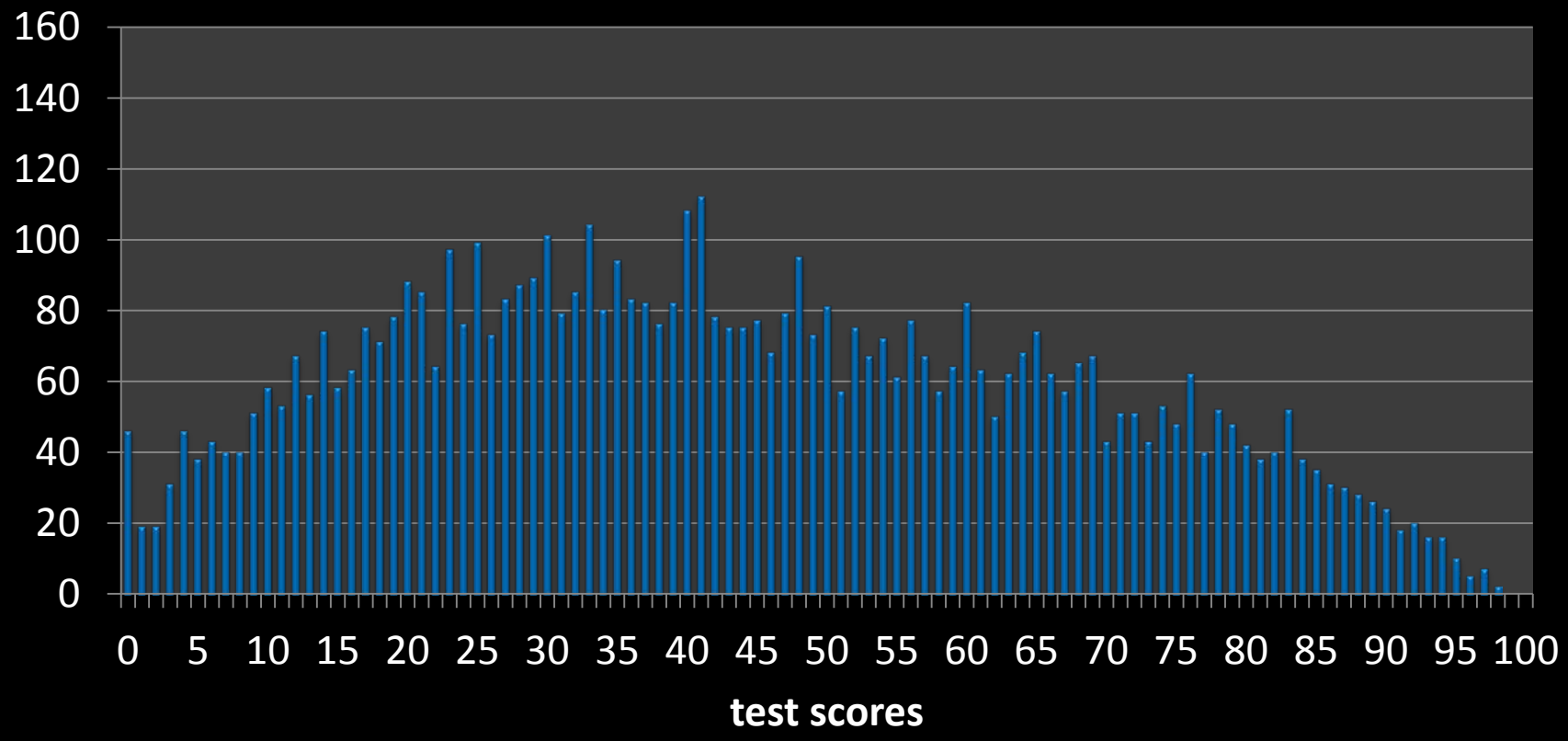
- Sampling distributions
- Detecting impact
  - significance
  - effect size
  - power
  - baseline and covariates
  - clustering
  - stratification

Baseline test scores



We implement the Balsakhi Program

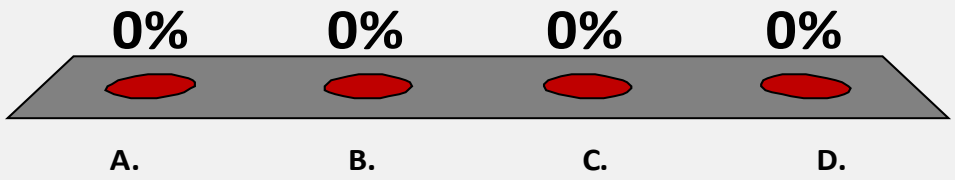
Endline test scores



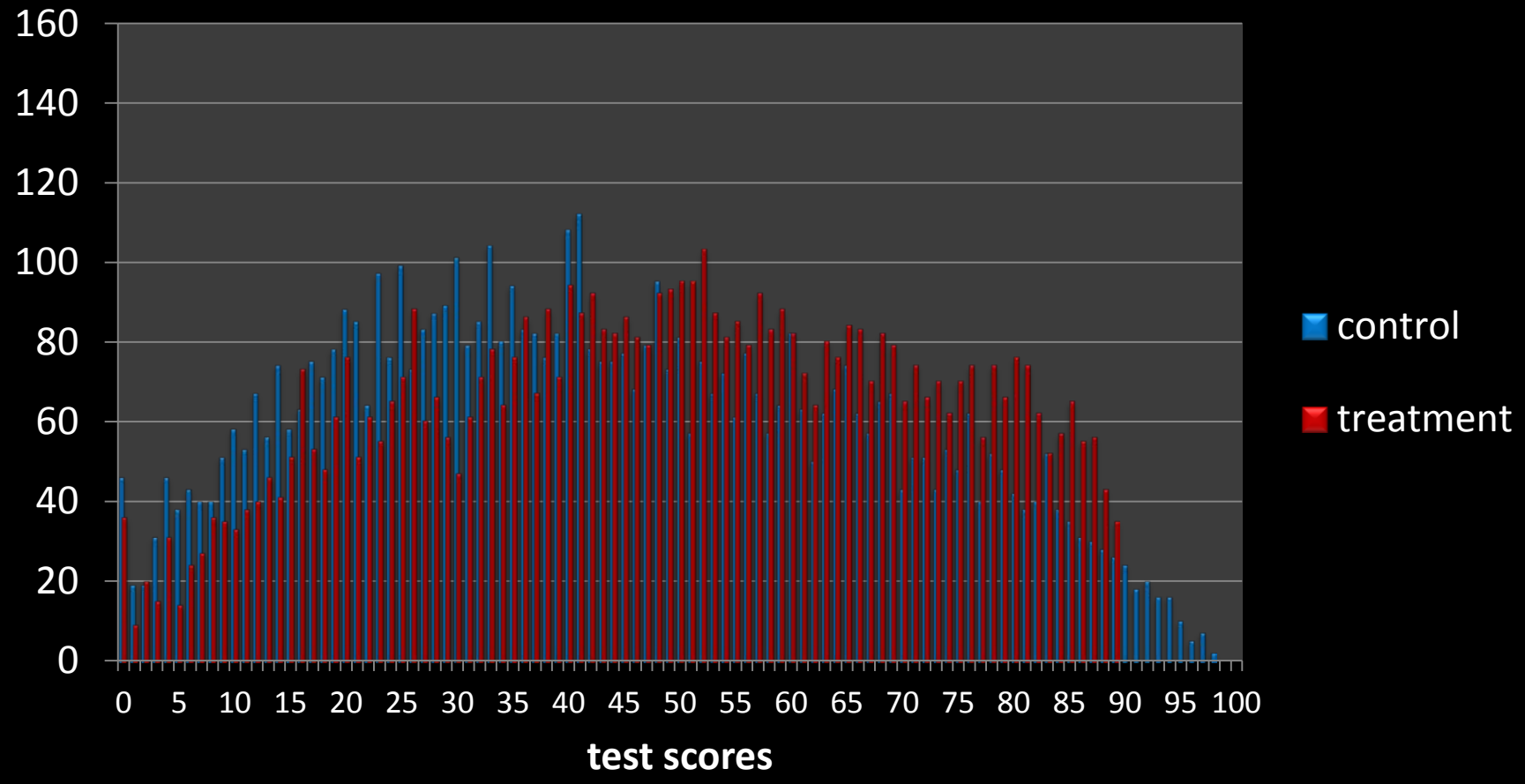
After the balsakhi programs, these are the endline test scores

The impact appears to be?

A. Positive
B. Negative
C. No impact
D. Don't know

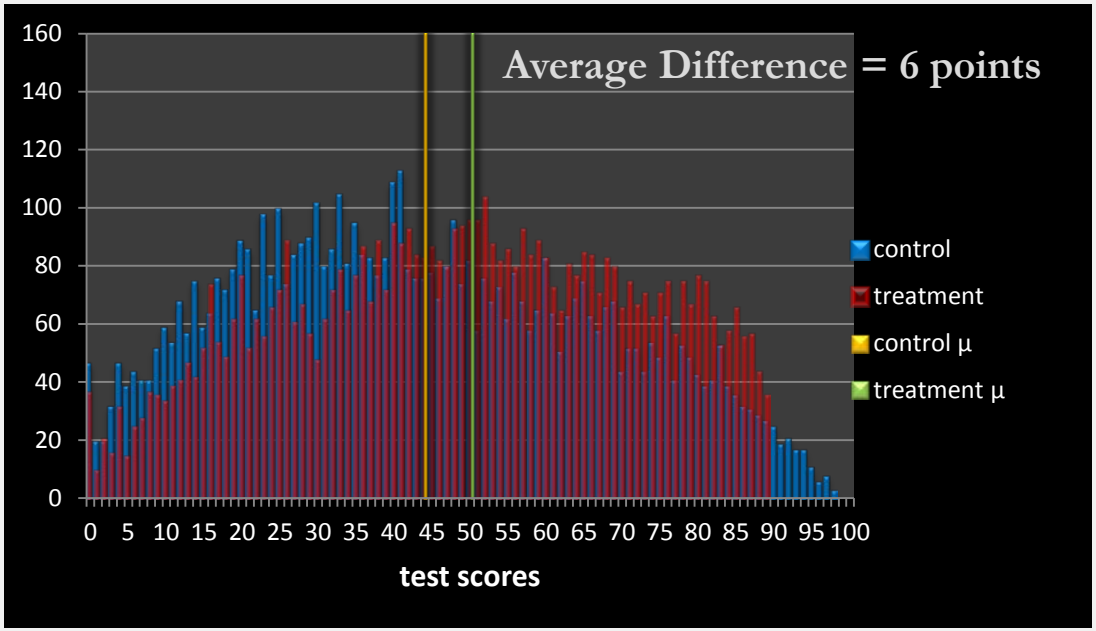


Post-test: control & treatment



- Stop! That was the control group. The treatment group is red.

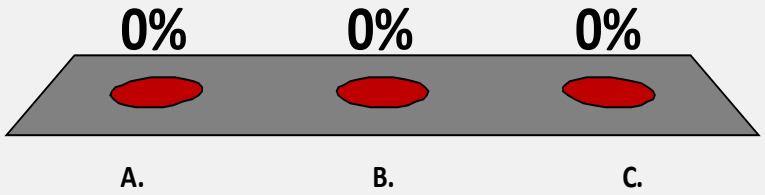
Is this impact statistically significant?



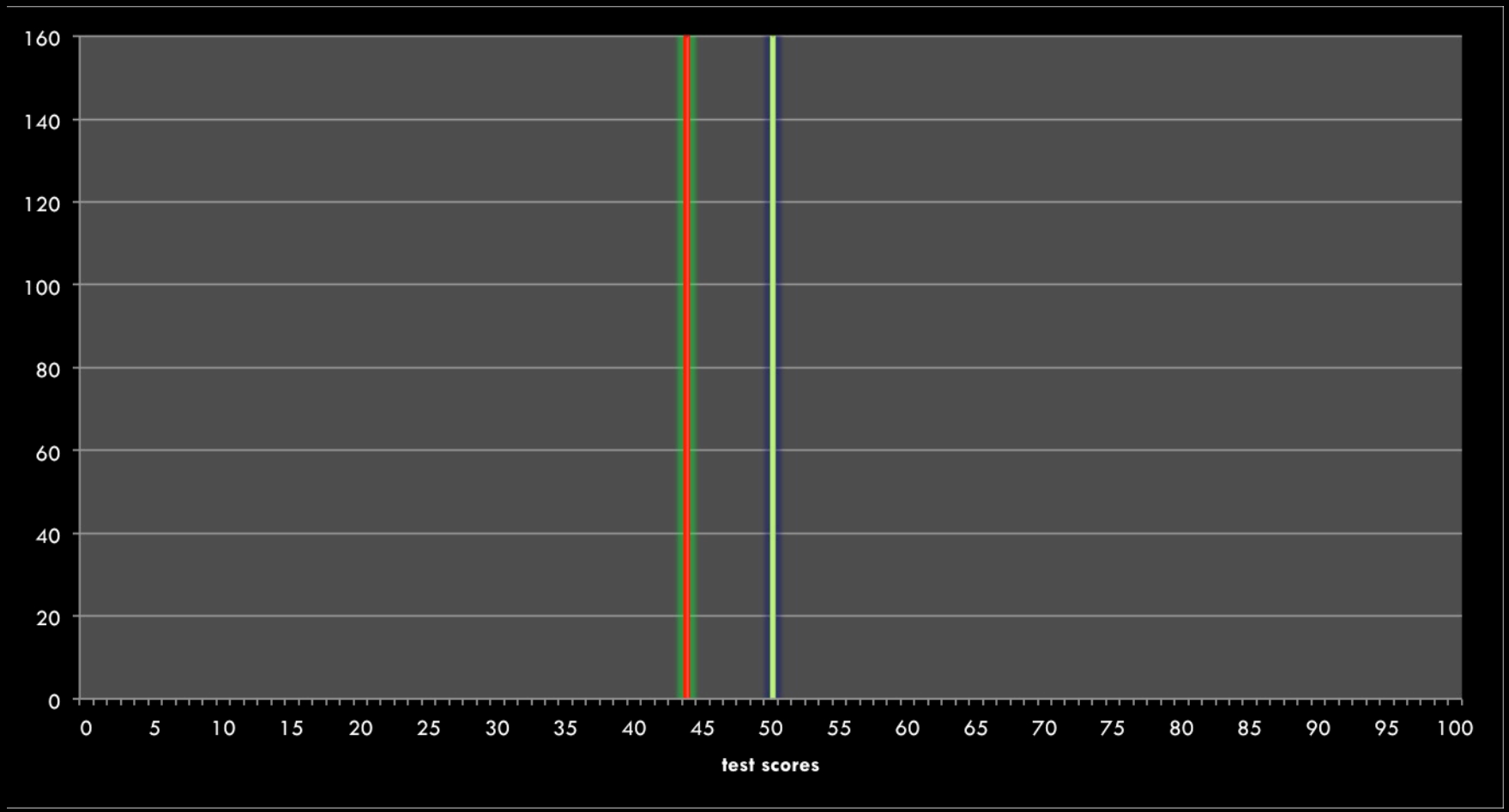
A. Yes

B. No

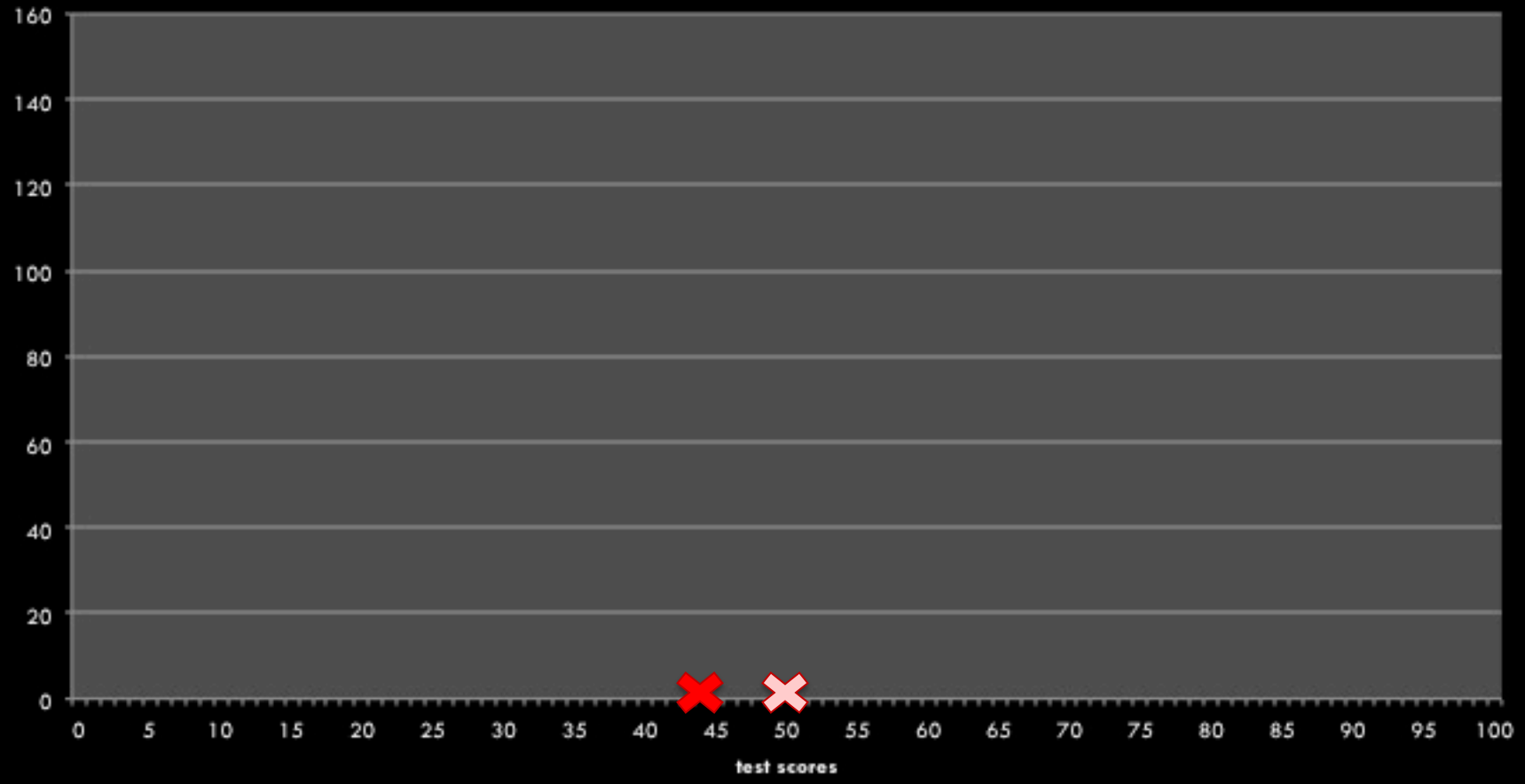
C. Don't know



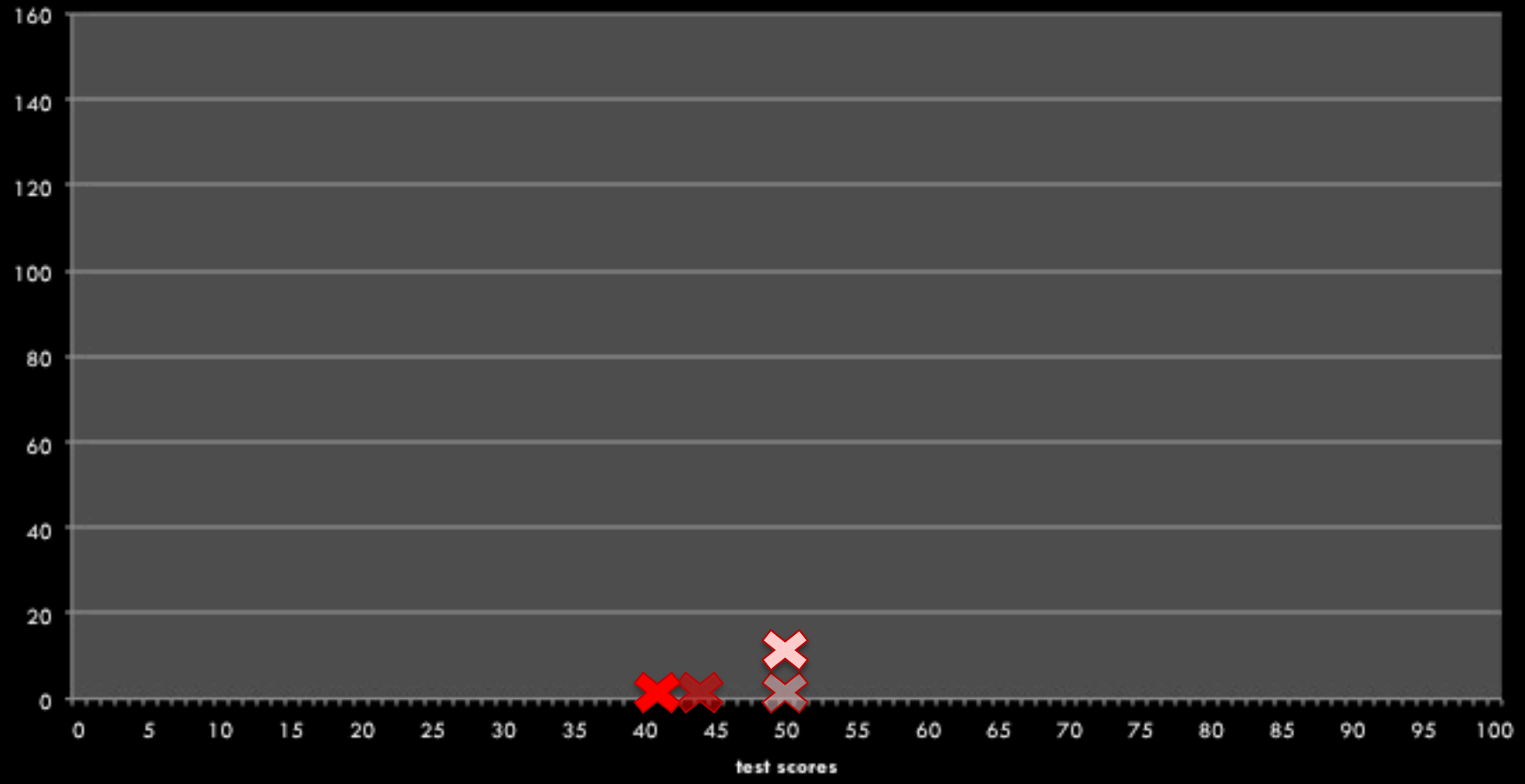
One experiment: 6 points



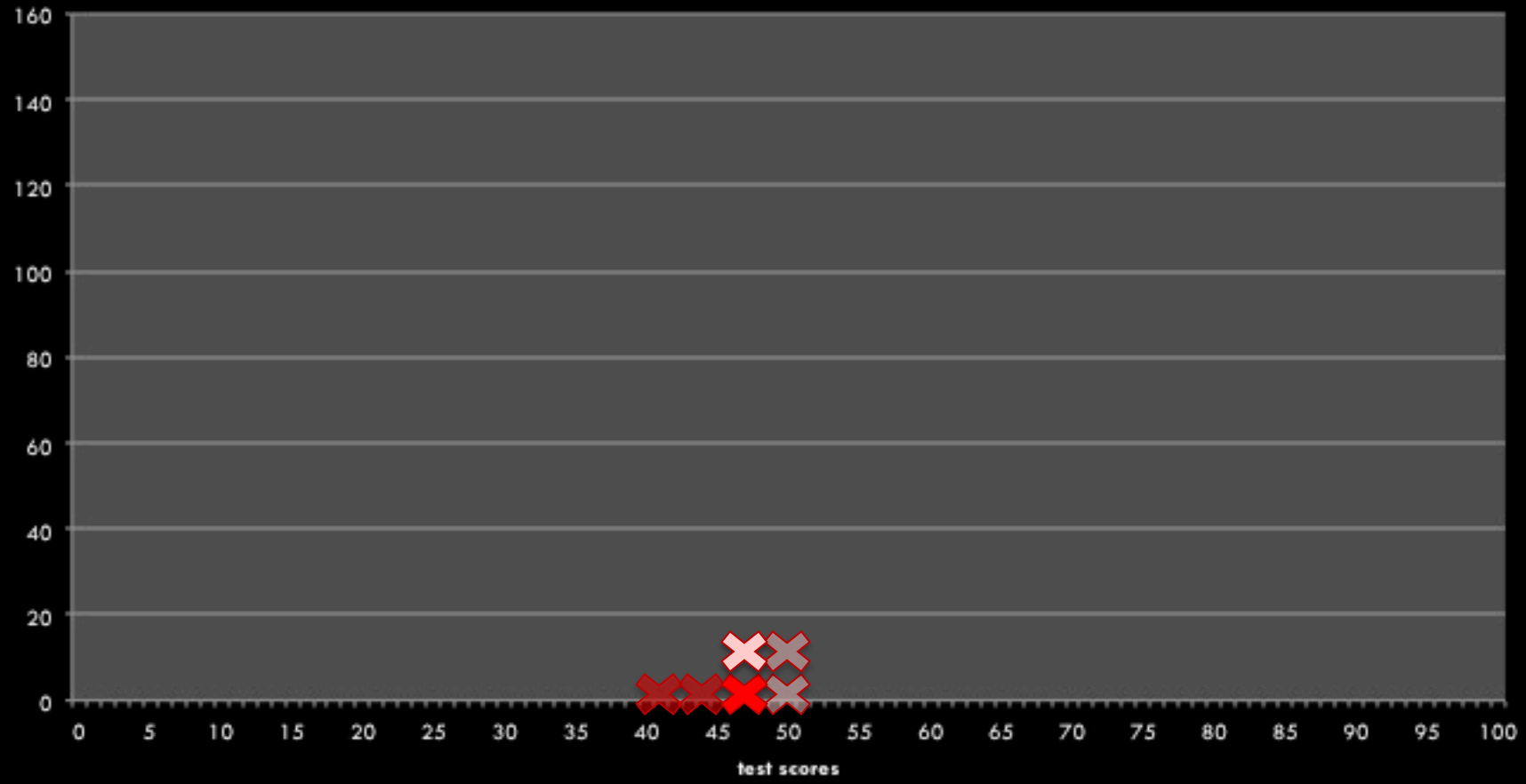
One experiment



Two experiments



A few more…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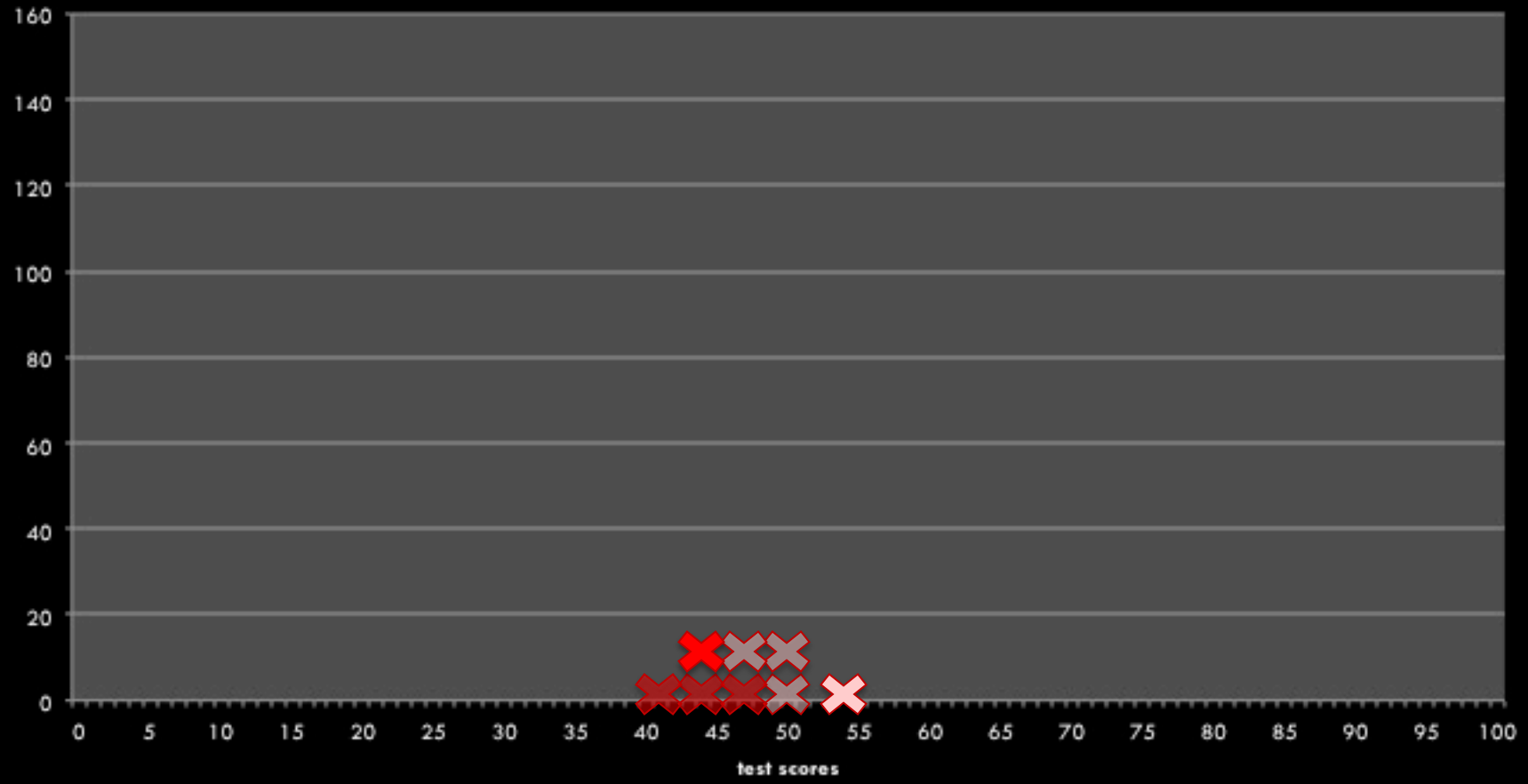
test scores

A few more…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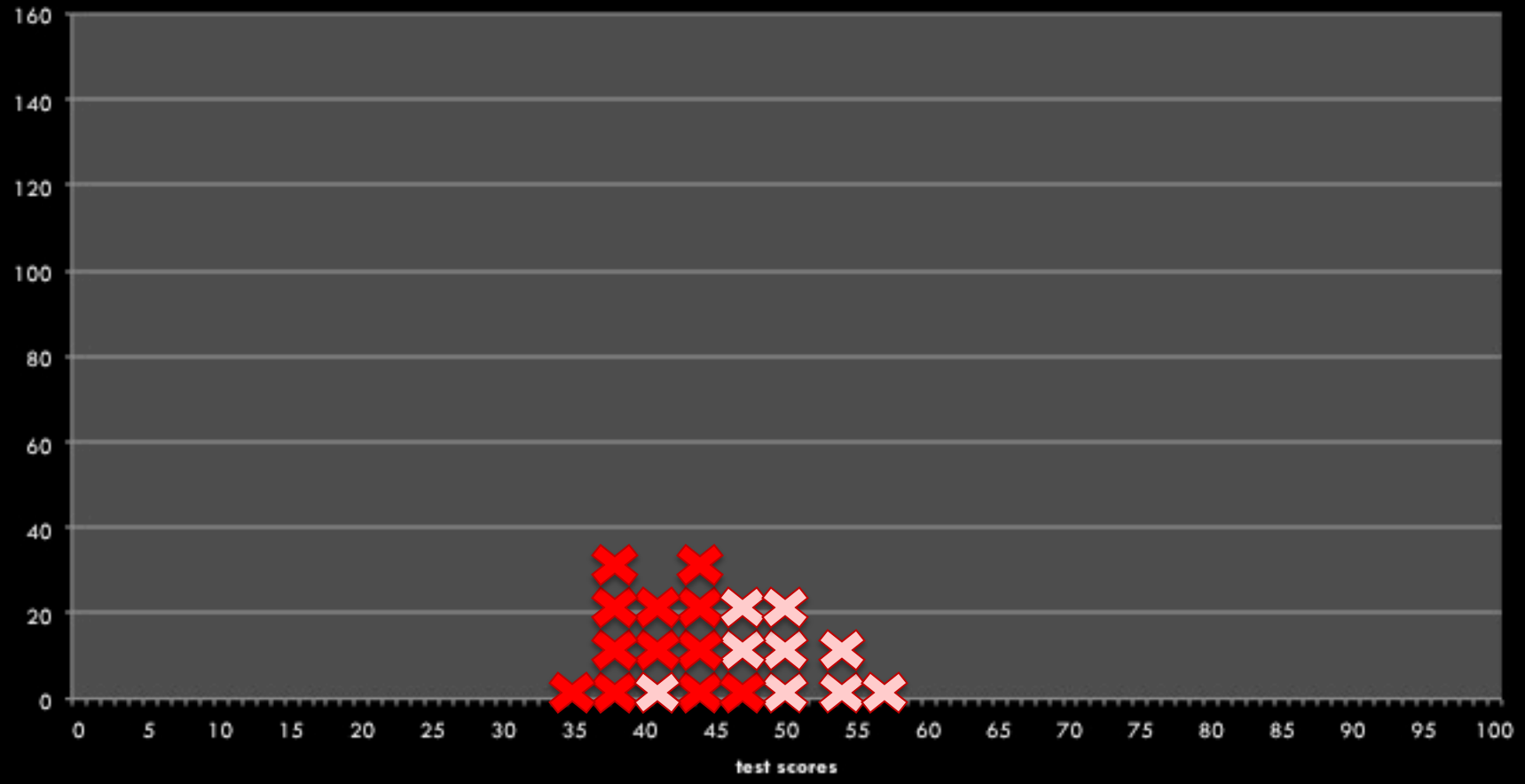
test scores

Many more…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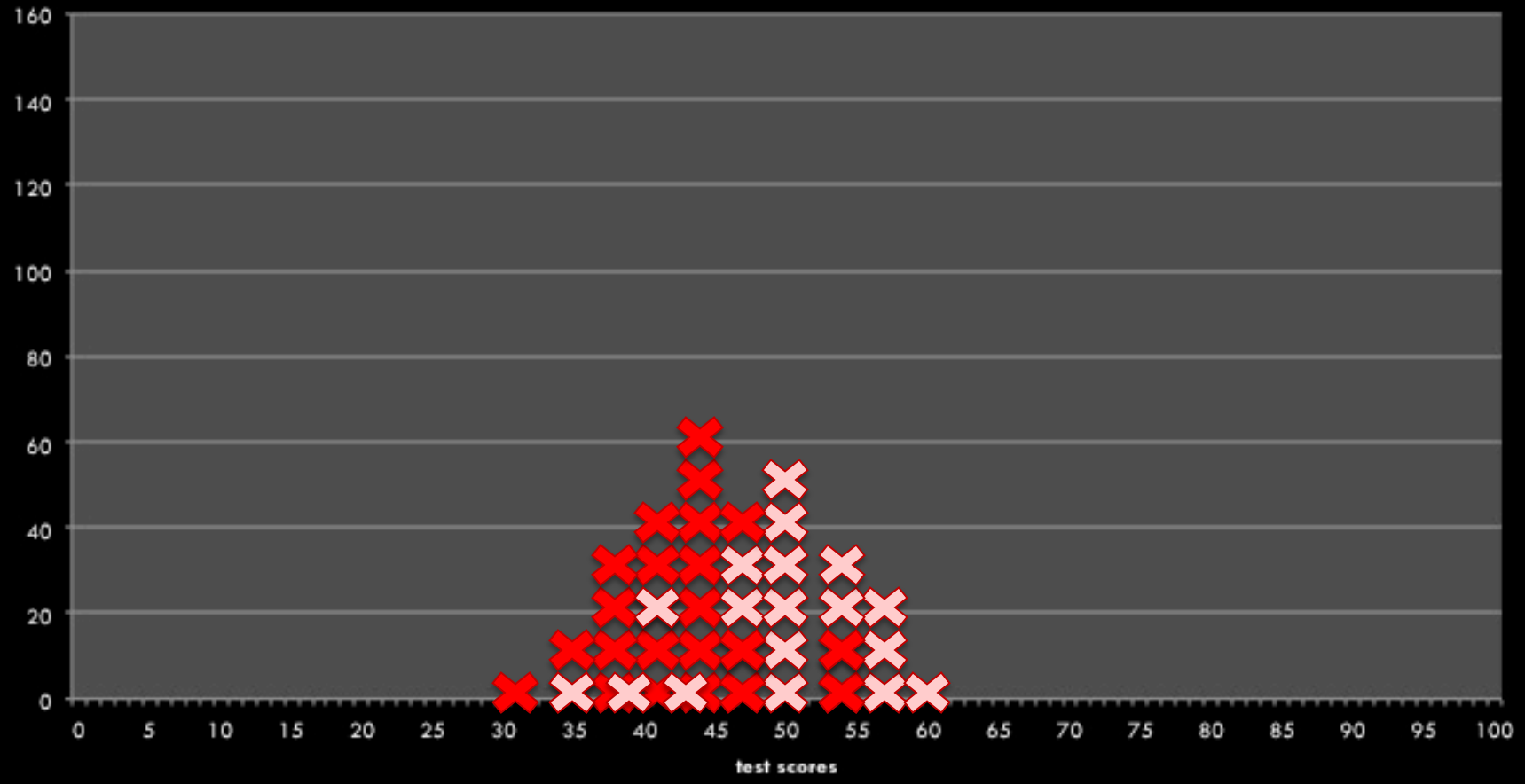
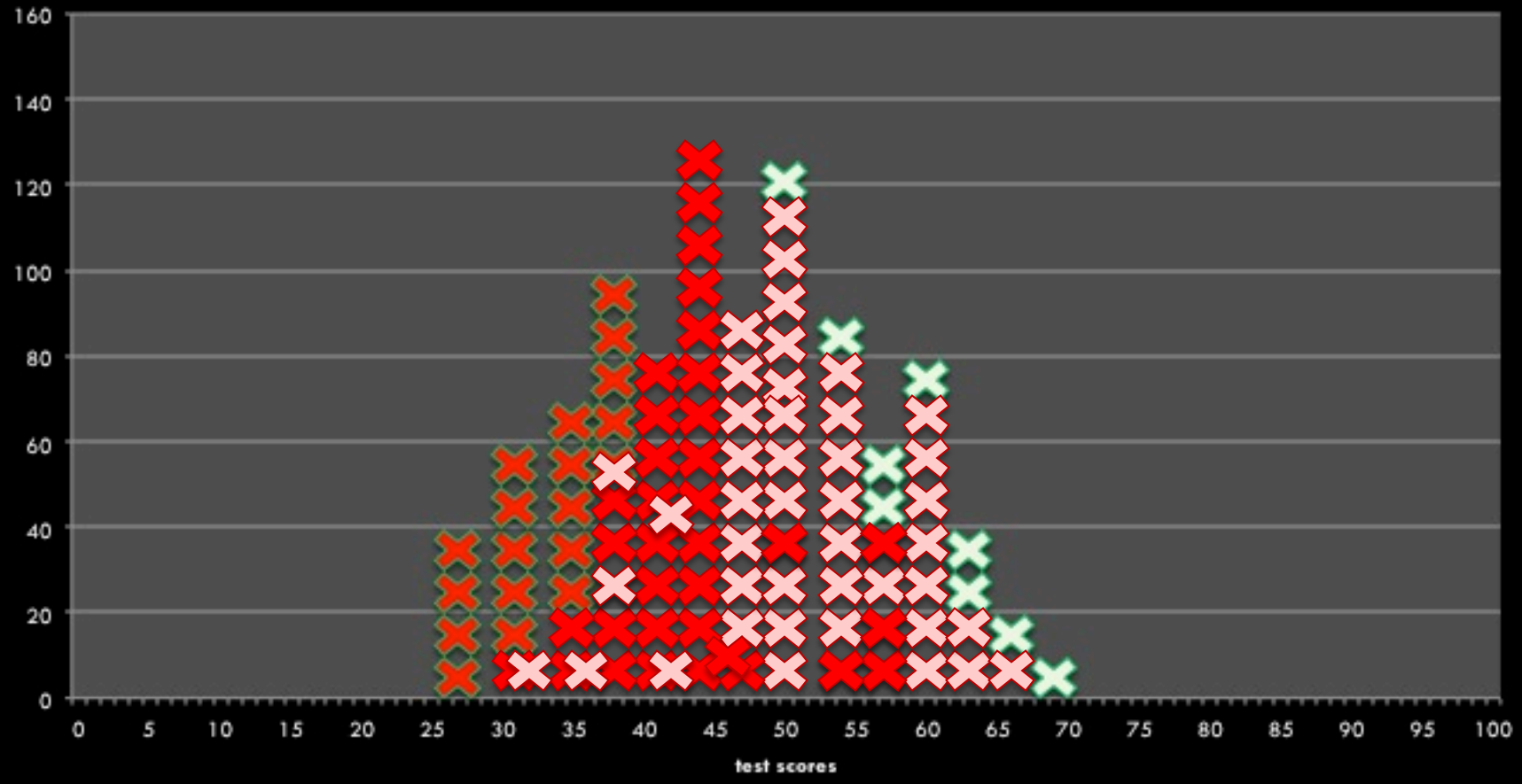
test scores

A whole lot more…

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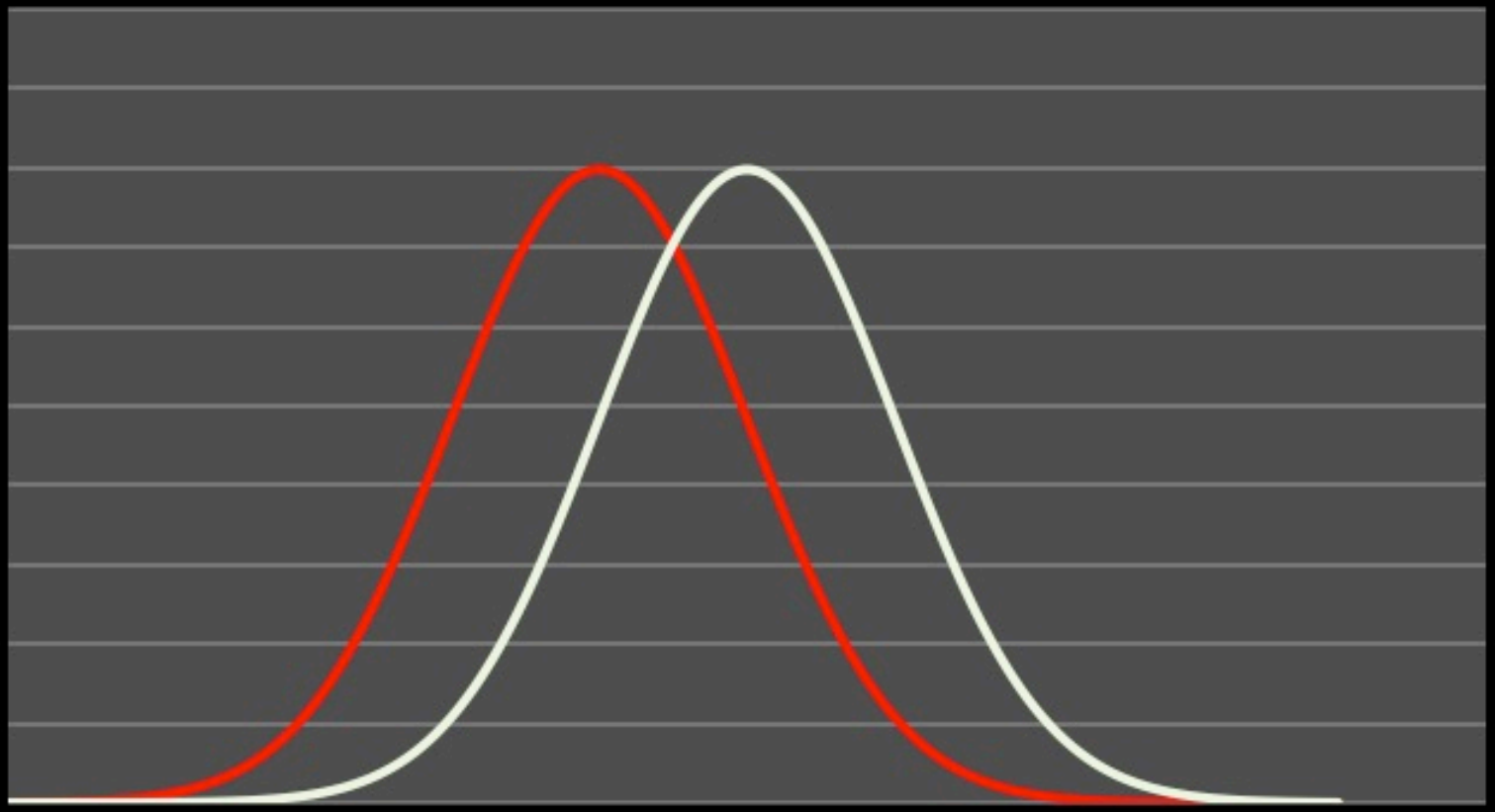
test scores

Running the experiment thousands of times…



By the Central Limit Theorem, these are normally distributed

Hypothesis testing

- In criminal law, most institutions follow the rule: "innocent until proven guilty"
- The presumption is that the accused is innocent and the burden is on the prosecutor to show guilt
  - The jury or judge starts with the "null hypothesis" that the accused person is innocent
  - The prosecutor has a hypothesis that the accused person is guilty

Hypothesis testing

- In program evaluation, instead of "presumption of innocence," the rule is: "presumption of insignificance"
- The "Null hypothesis" (H_0) is that there was no (zero) impact of the program
- The burden of proof is on the evaluator to show a significant effect of the program

Hypothesis testing: conclusions

- If it is very unlikely (less than a 5% probability) that the difference is solely due to chance:
  - We "reject our null hypothesis"
- We may now say:
  - "our program has a statistically significant impact"

What is the significance level?

- Type I error: rejecting the null hypothesis even though it is true (false positive)
- Significance level: *The probability* that we will reject the null hypothesis even though it is true

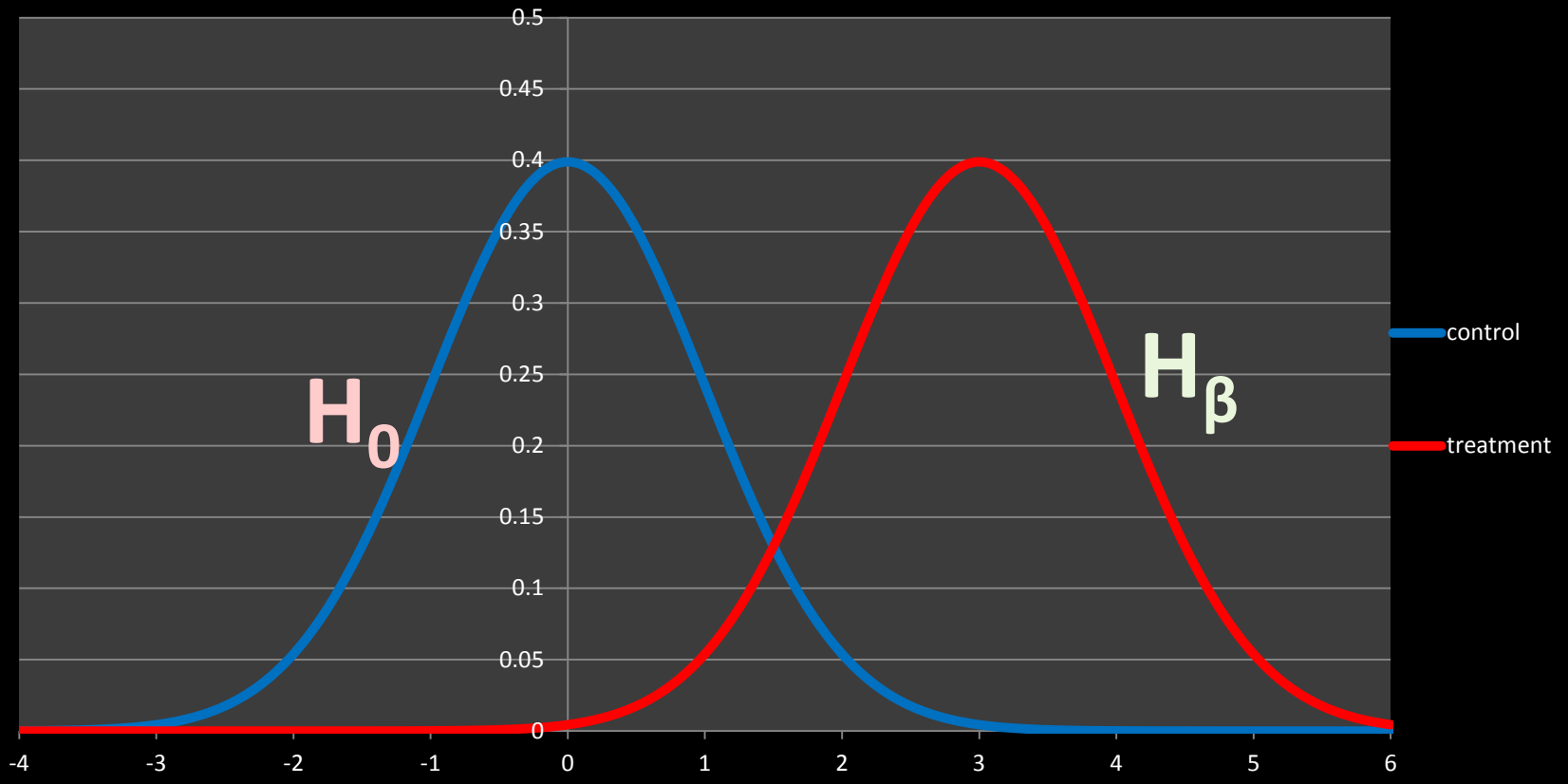
Hypothesis testing: 95% confidence

		YOU CONCLUDE	
		Effective	*No Effect*
THE TRUTH	*Effective*	☺	**Type II Error** (low power) ☹
	No Effect	**Type I Error** (5% of the time) ☹	☺

What is Power?

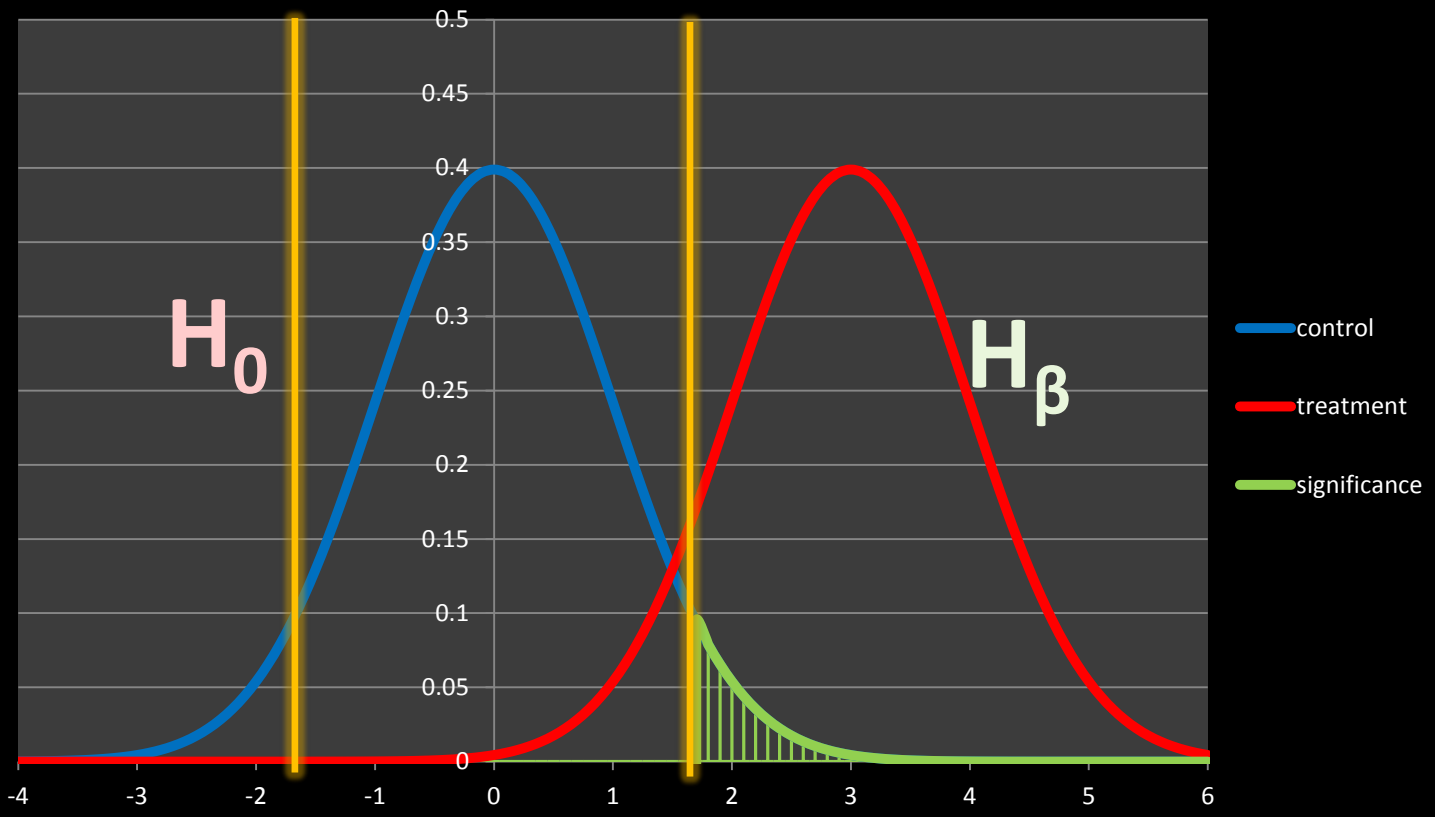
- Type II Error: Failing to reject the null hypothesis (concluding there is no difference), when indeed the null hypothesis is false.
- Power: If there is a measureable effect of our intervention (the null hypothesis is false), the probability that we will detect an effect (reject the null hypothesis)

Before the experiment



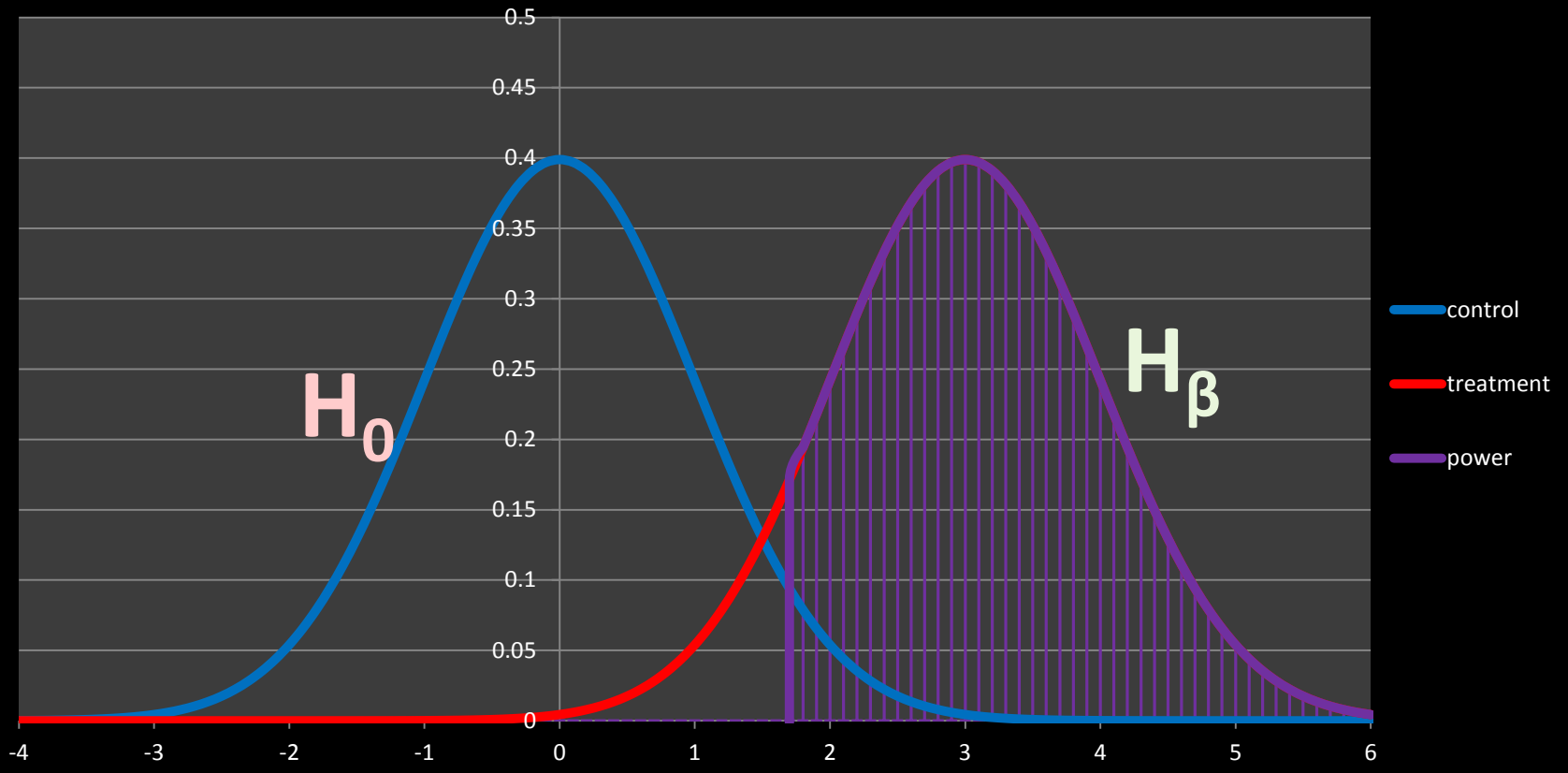
- Assume two effects: no effect and treatment effect β

Impose significance level of 5%



Anything between lines cannot be distinguished from 0

Can we distinguish Hβ from H0 ?



Shaded area shows % of time we would find Hβ true if it was

What influences power?

- What are the factors that change the proportion of the research hypothesis that is shaded—i.e. the proportion that falls to the right (or left) of the null hypothesis curve?
- Understanding this helps us design more powerful experiments

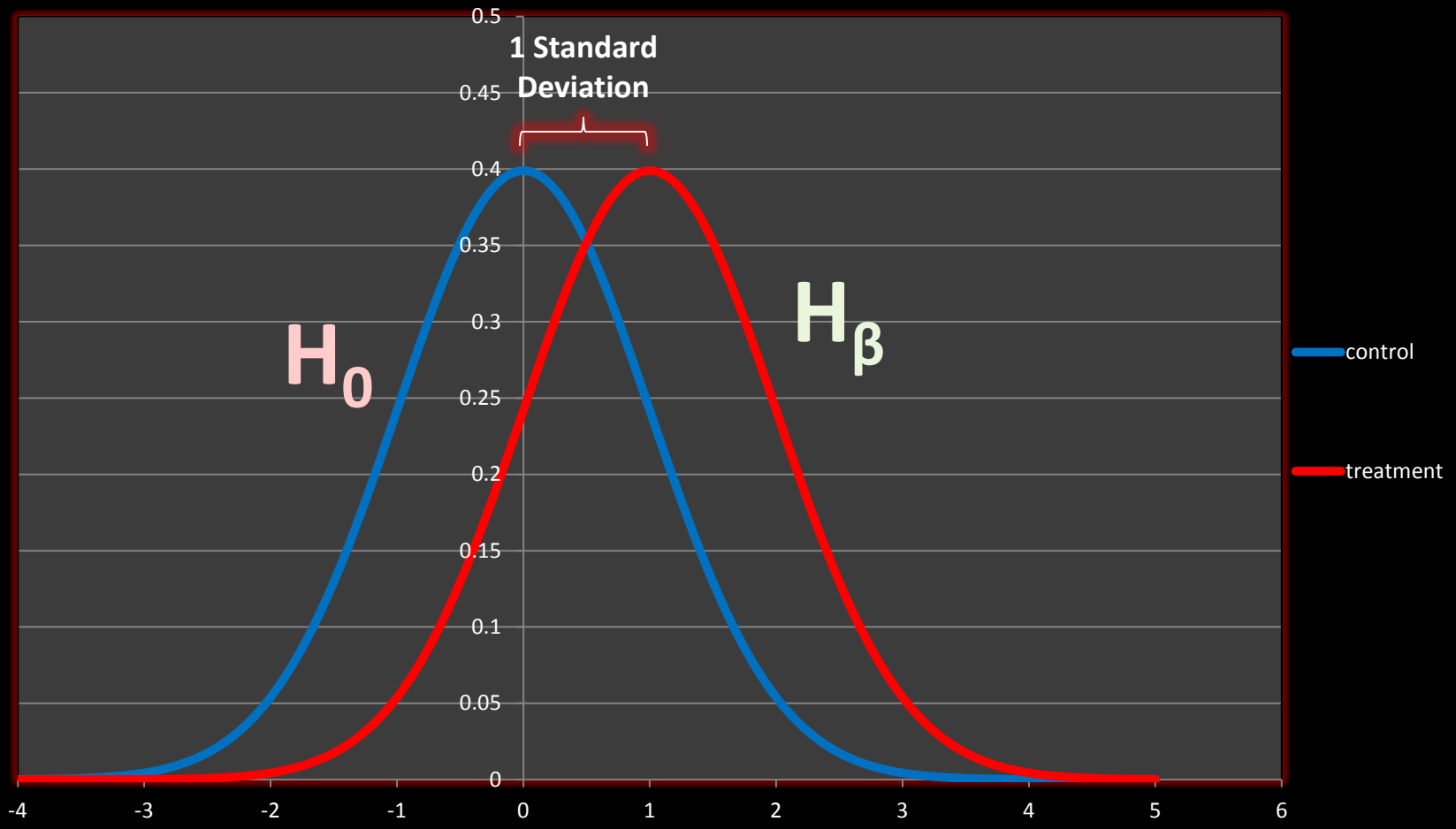
Power: main ingredients

1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

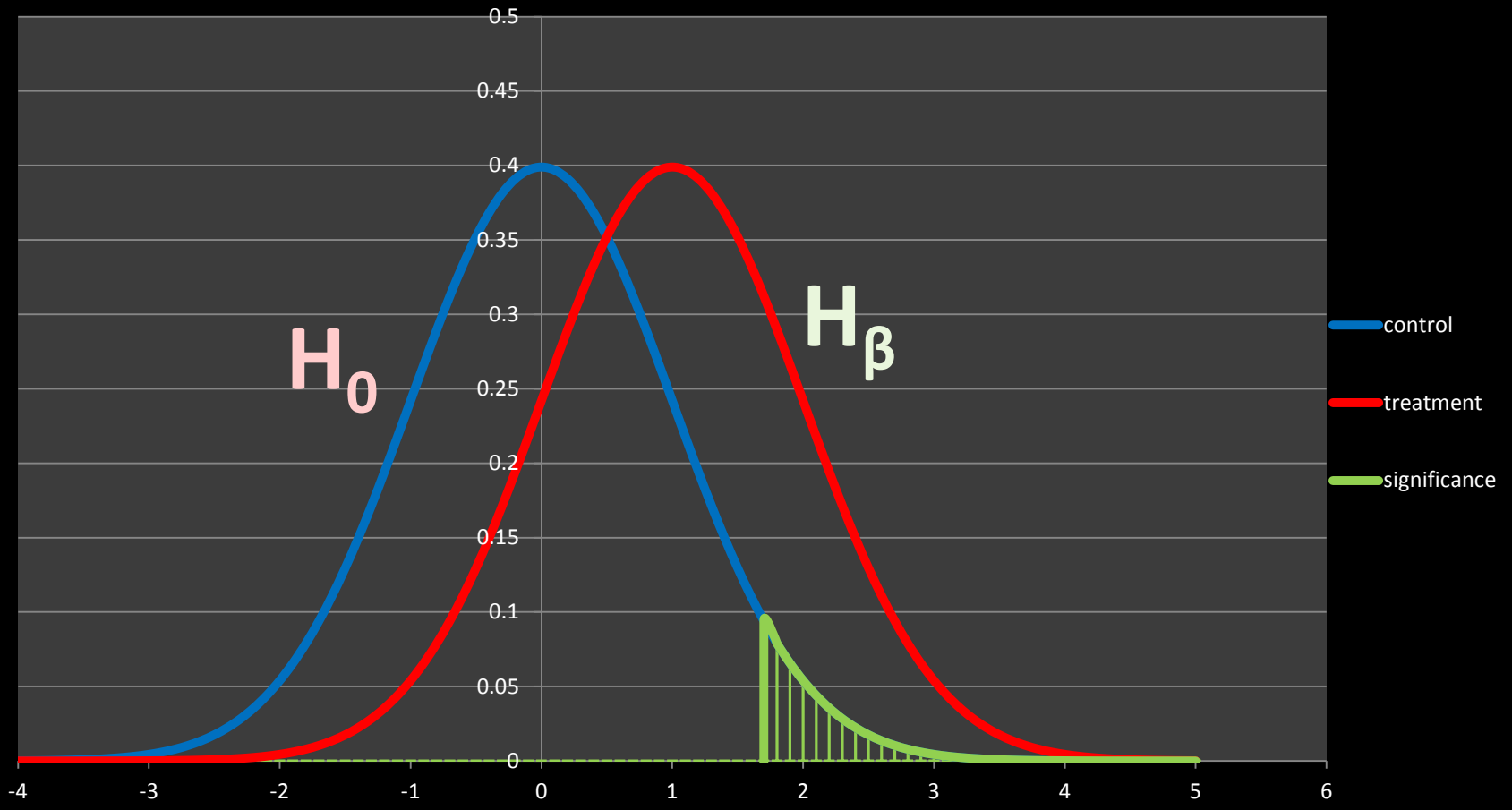
Power: main ingredients

1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

Effect Size: 1*SE

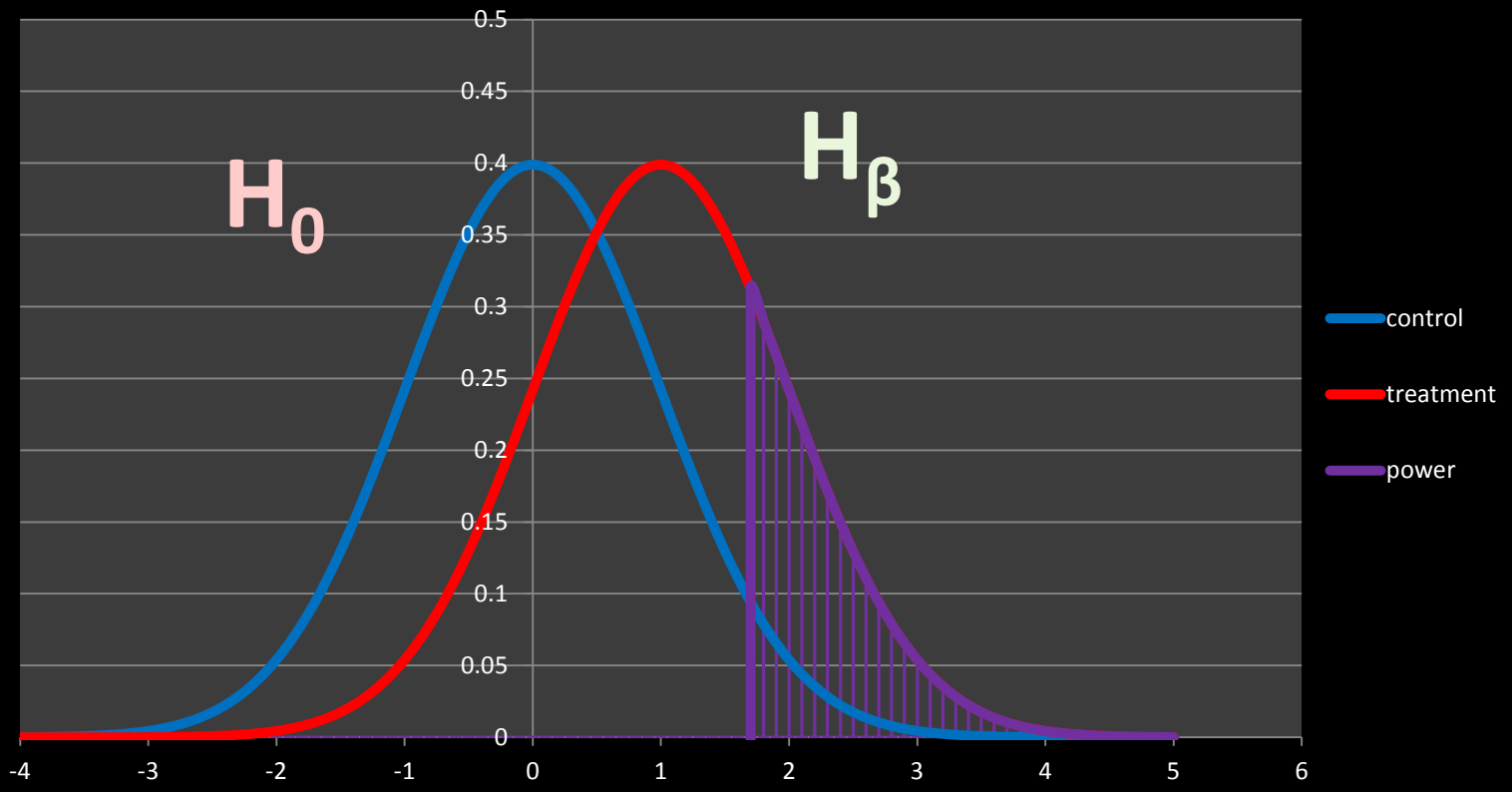


Effect Size = 1*SE



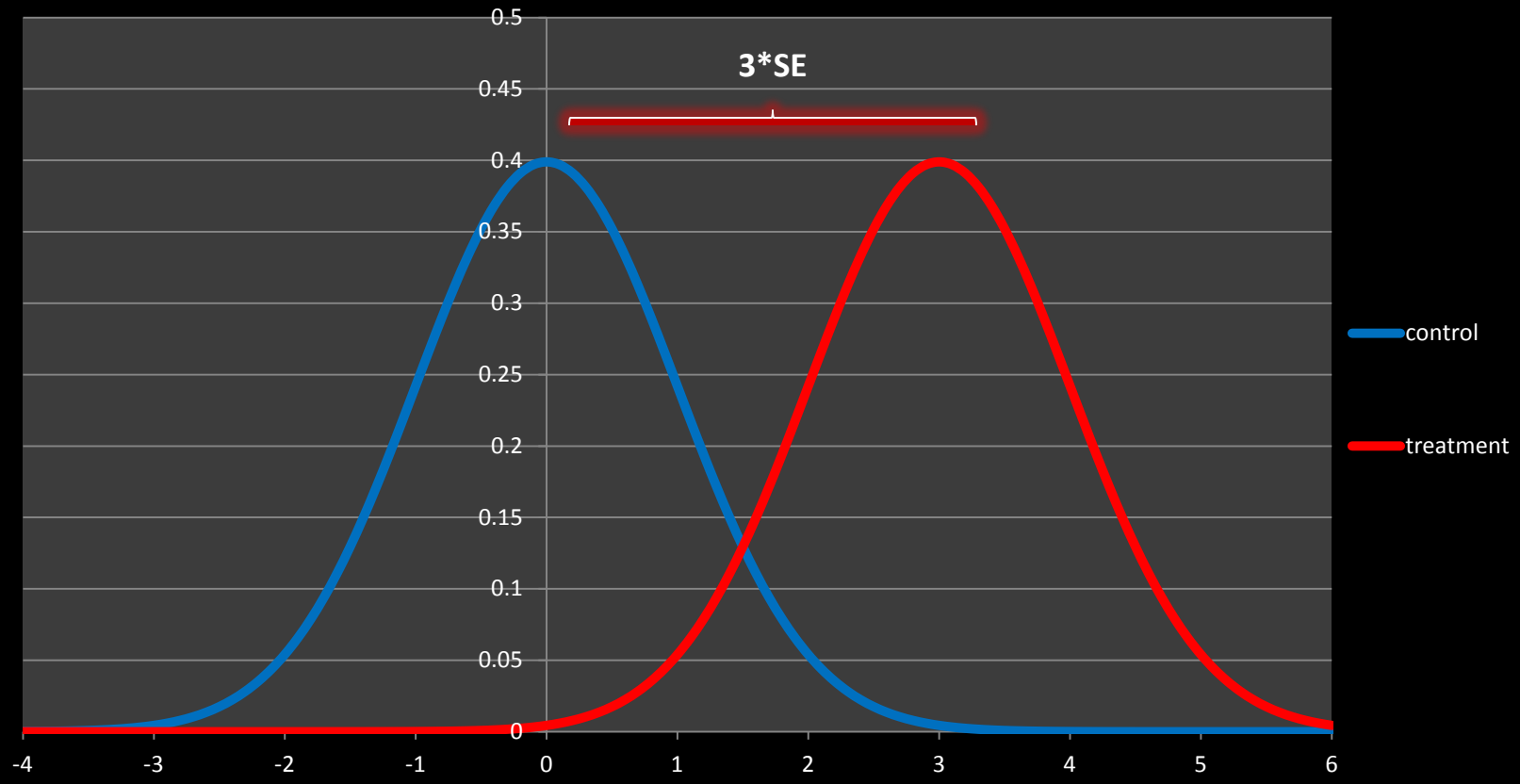
Power: 26%

If the true impact was 1*SE…



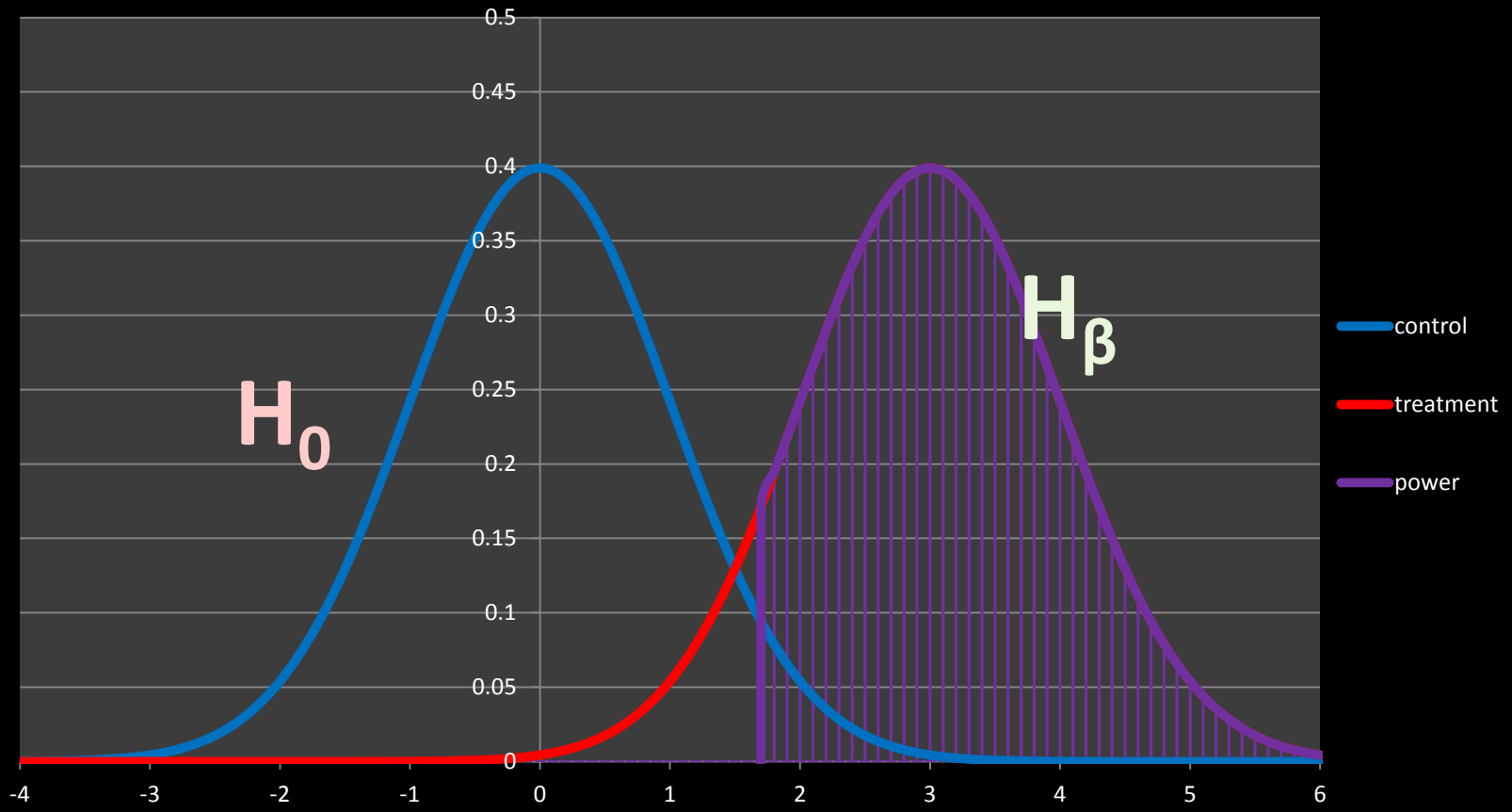
The Null Hypothesis would be rejected only 26% of the time

Effect Size: 3*SE



Bigger hypothesized effect size→ distributions farther apart

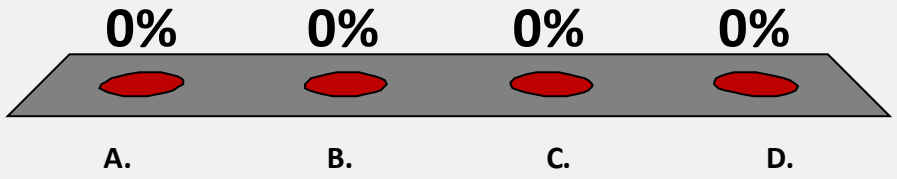
Effect size 3*SE: Power= 91%



Bigger Effect size means more power

What effect size should you use when designing your experiment?

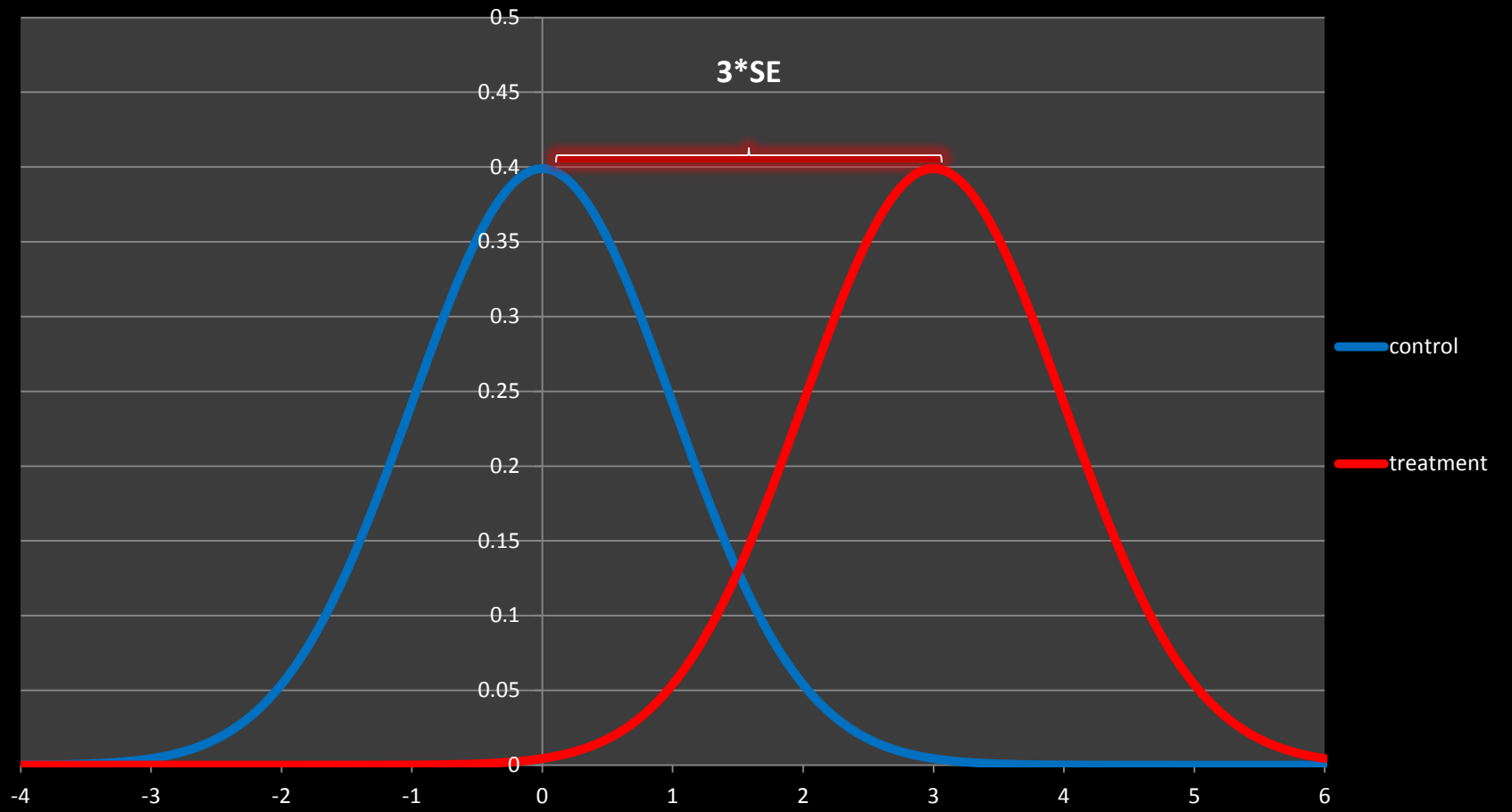
A. Smallest effect size that is still cost effective
B. Largest effect size you estimate your program to produce
C. Both
D. Neither



Effect size and take-up

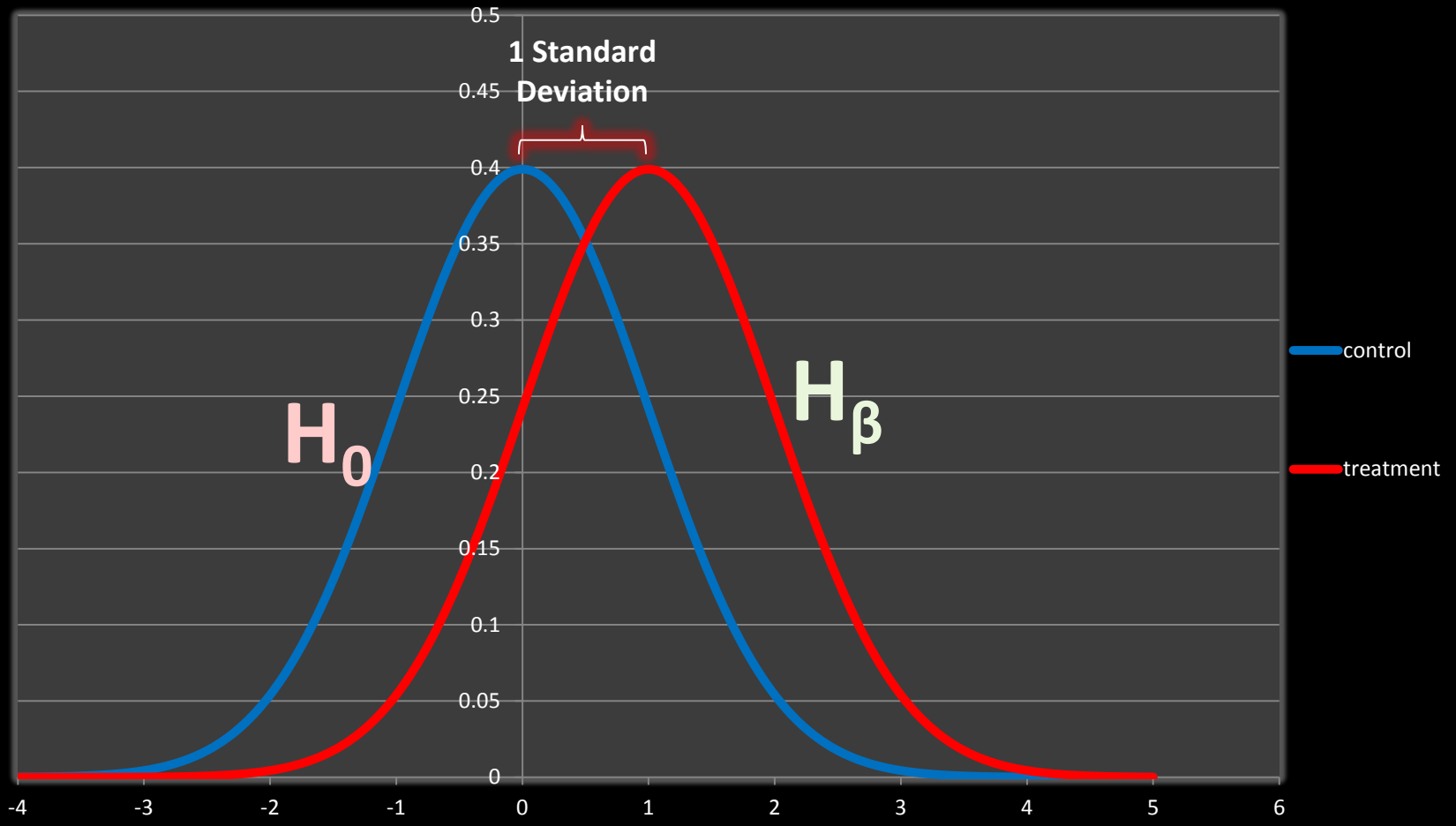
- Let's say we believe the impact on our participants is "3"
- What happens if take up is 1/3?
- Let's show this graphically

Effect Size: 3*SE

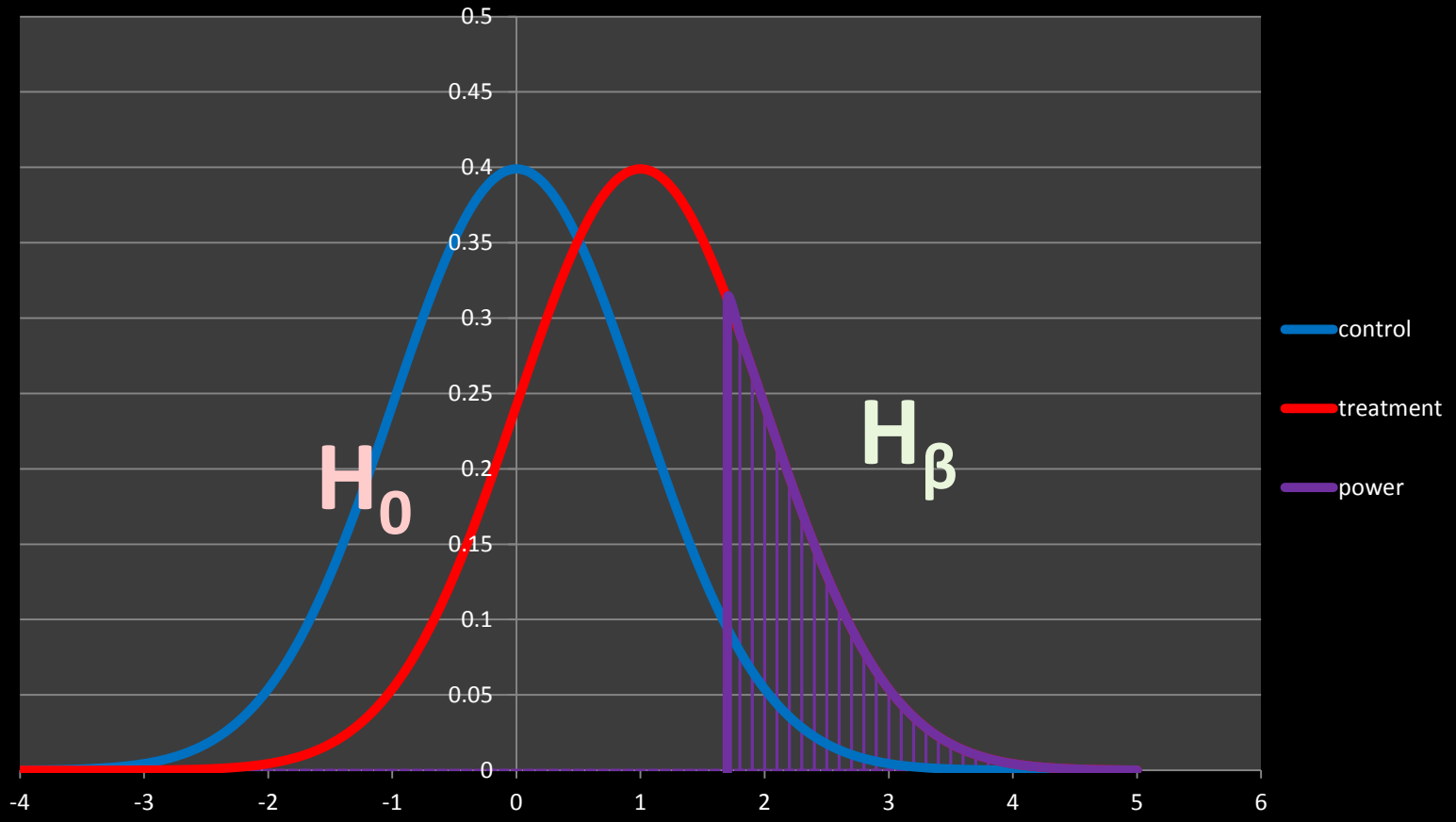


Let's say we believe the impact on our participants is "3"

Take up is 33%. Effect size is 1/3rd



Back to: Power = 26%

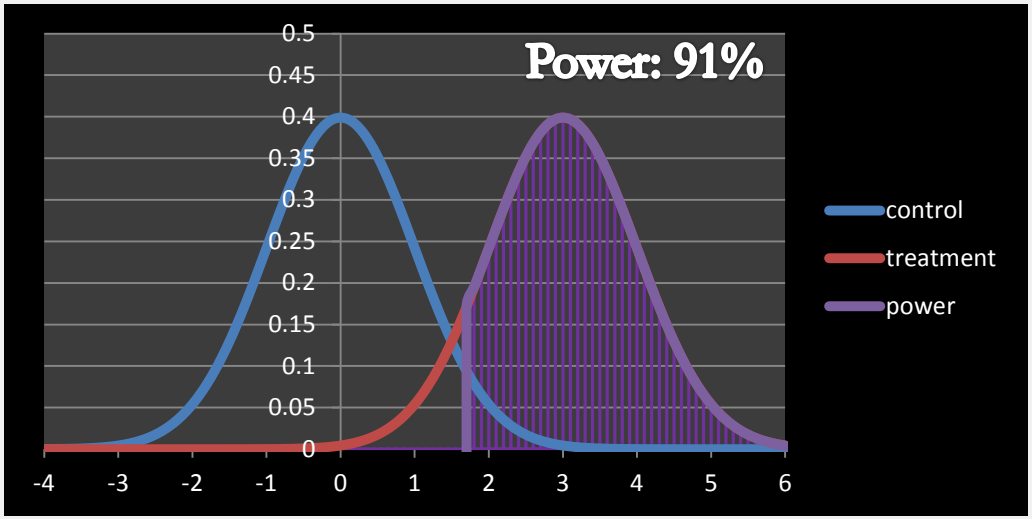


Take-up is reflected in the effect size

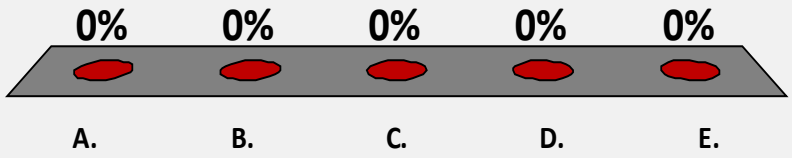
Power: main ingredients

1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

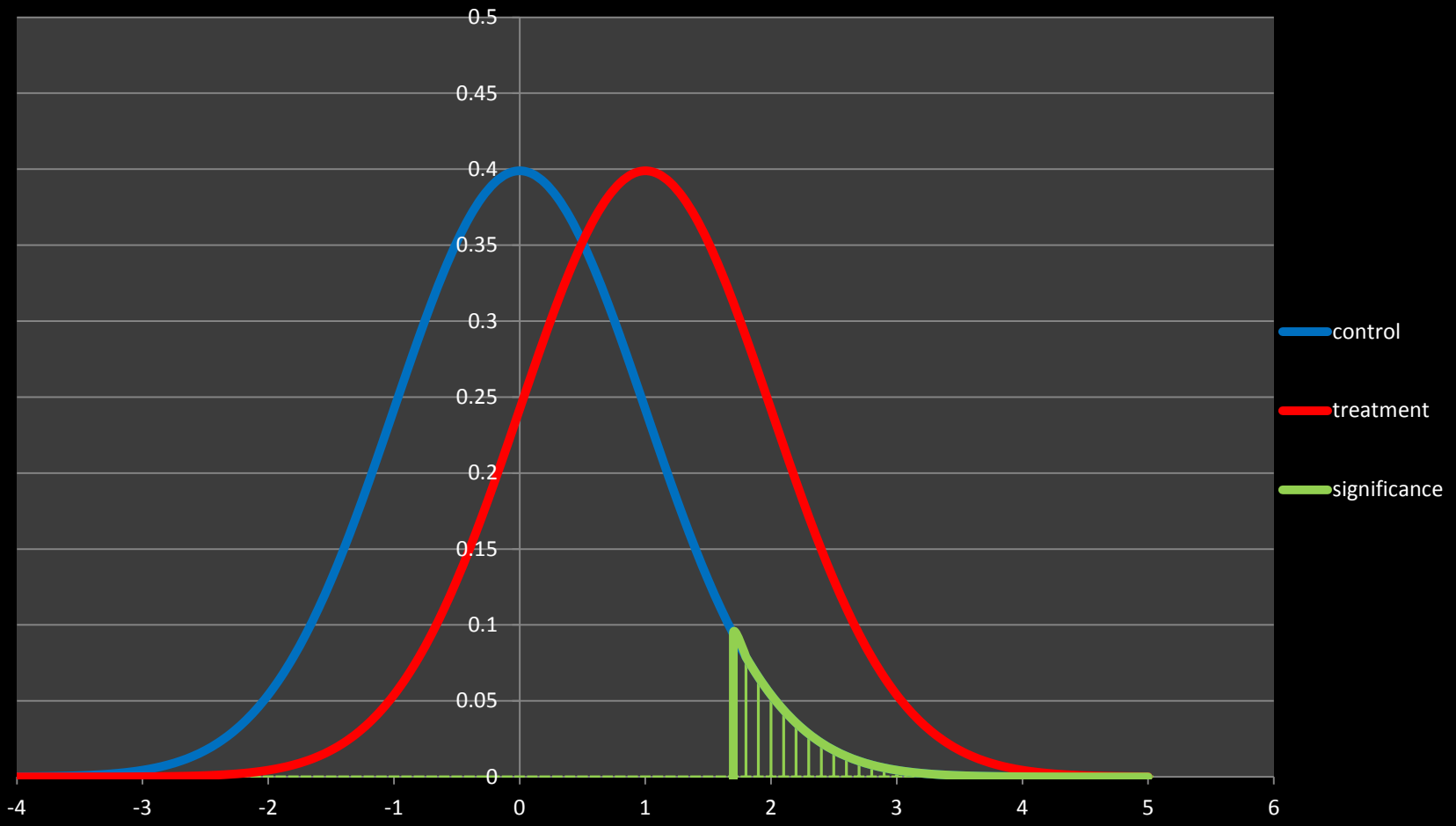
By increasing sample size you increase…



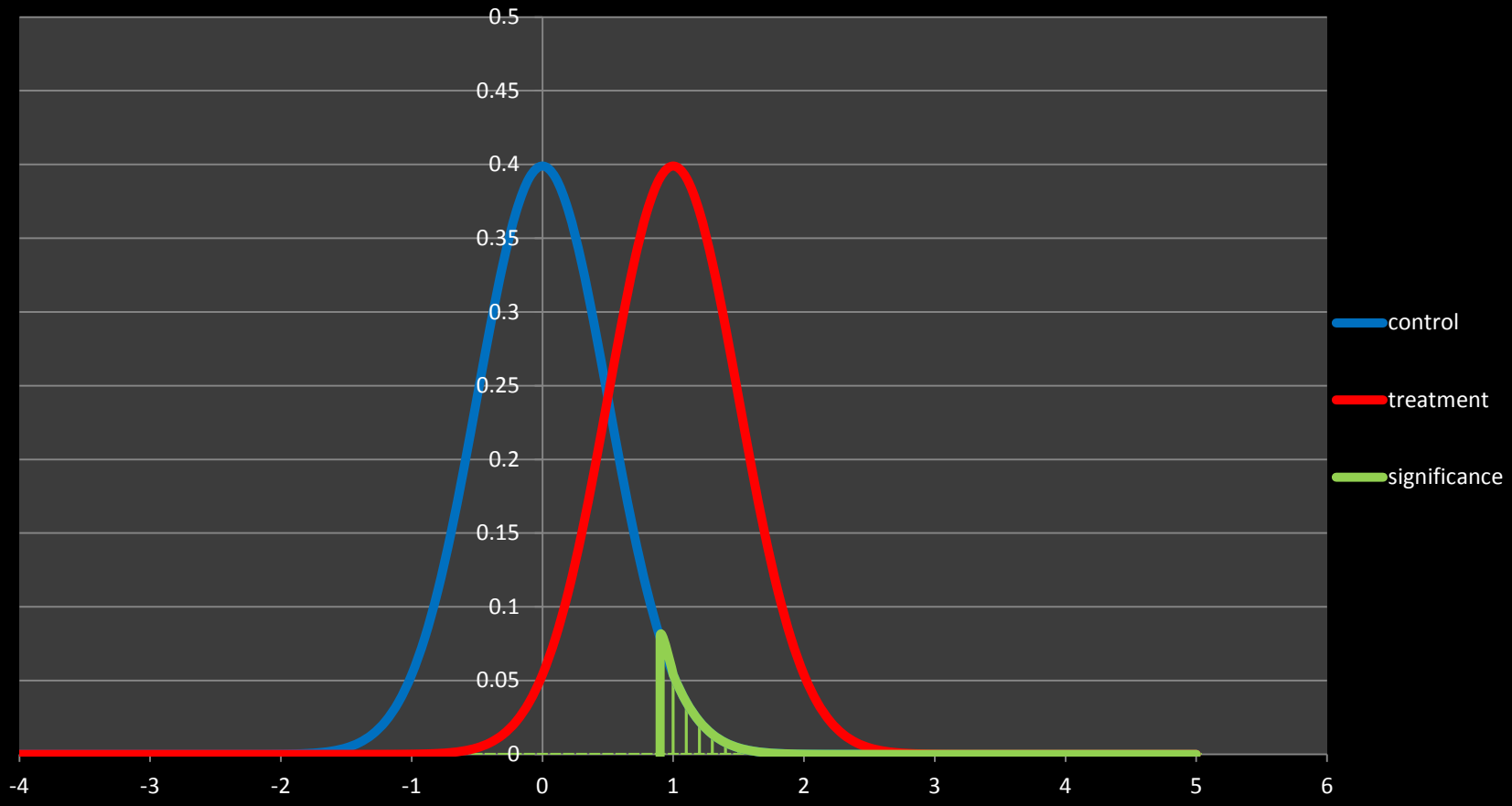
A. Accuracy
B. Precision
C. Both
D. Neither
E. Don't know



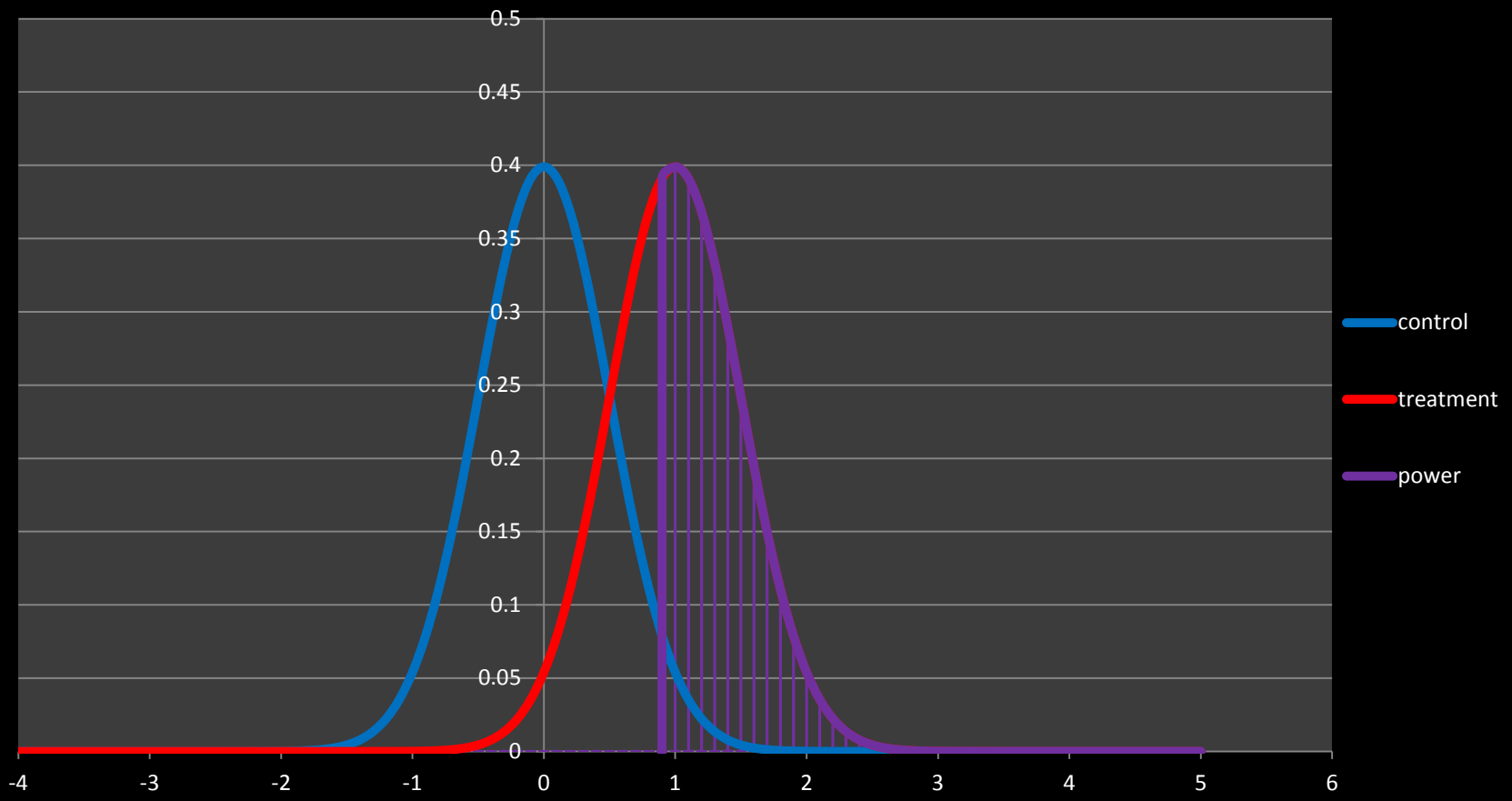
Power: Effect size = 1SD, Sample size = N



Power: Sample size = 4N



Power: 64%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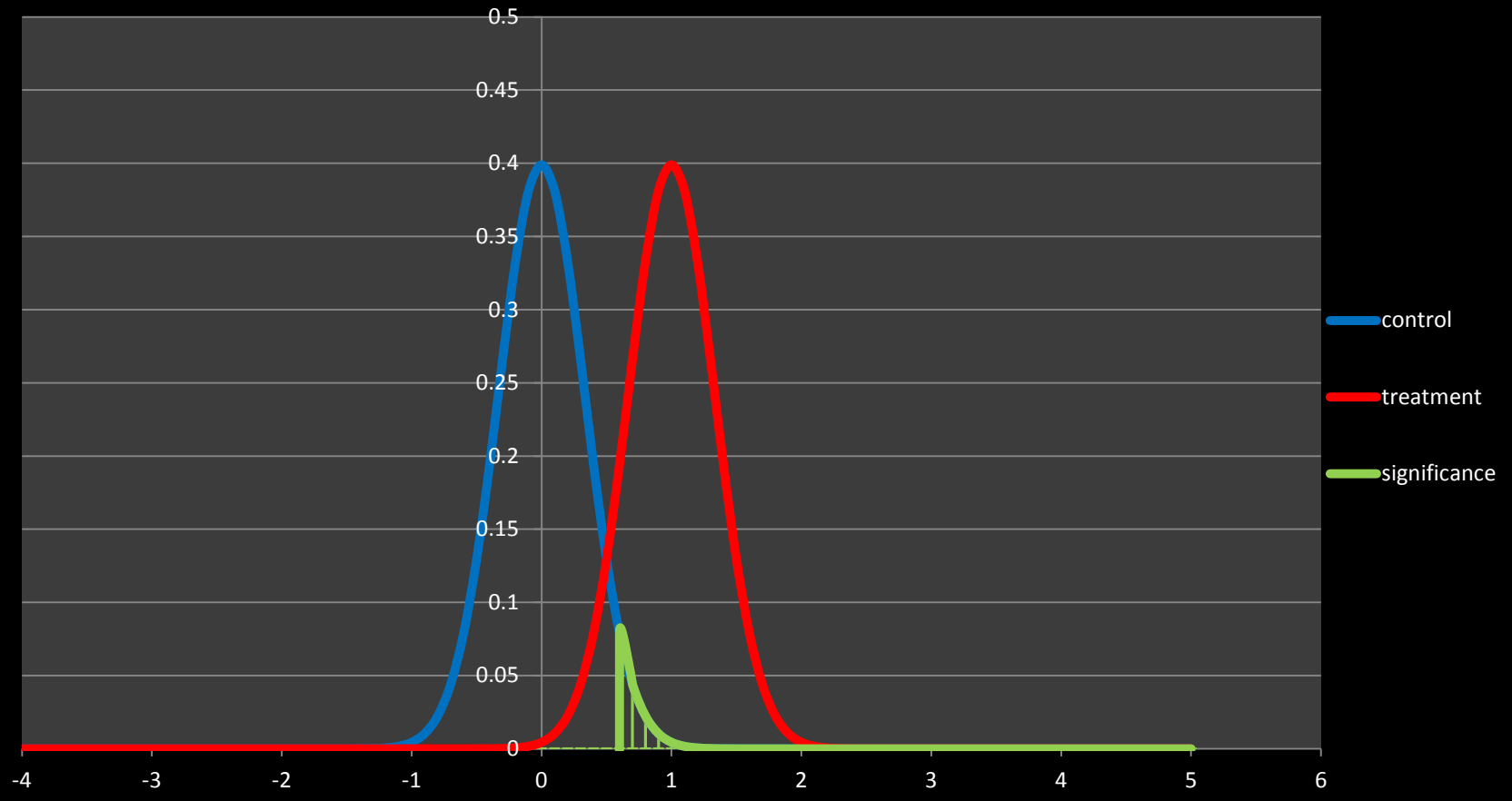
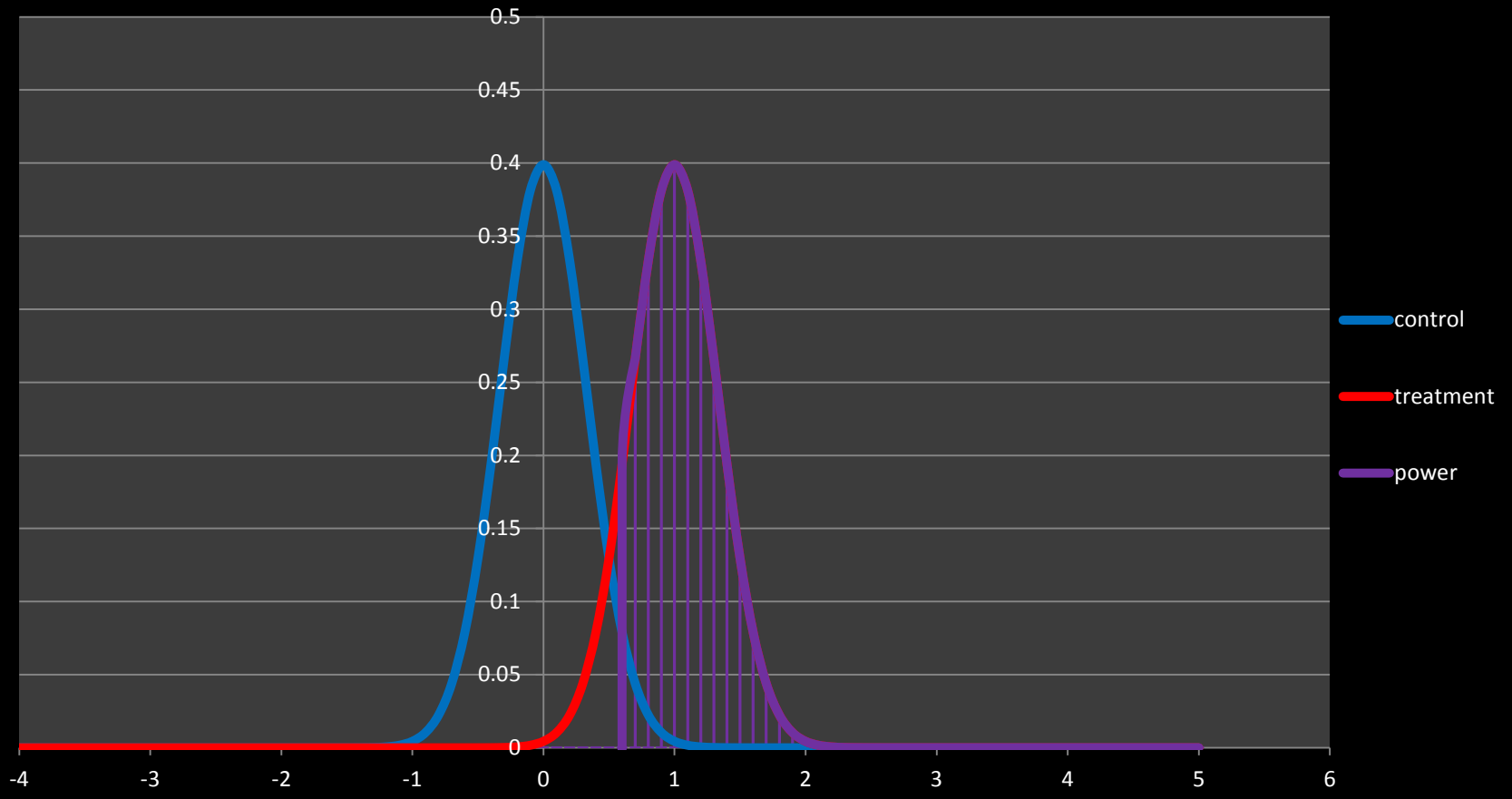
Power: Sample size = 9



Power: 91%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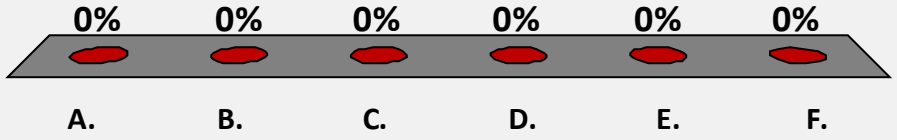
control
treatment
power

Power: main ingredients

1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

What are typical ways to reduce the underlying variance

A. Include covariates
B. Increase the sample
C. Do a baseline survey
D. All of the above
E. A and B
F. A and C



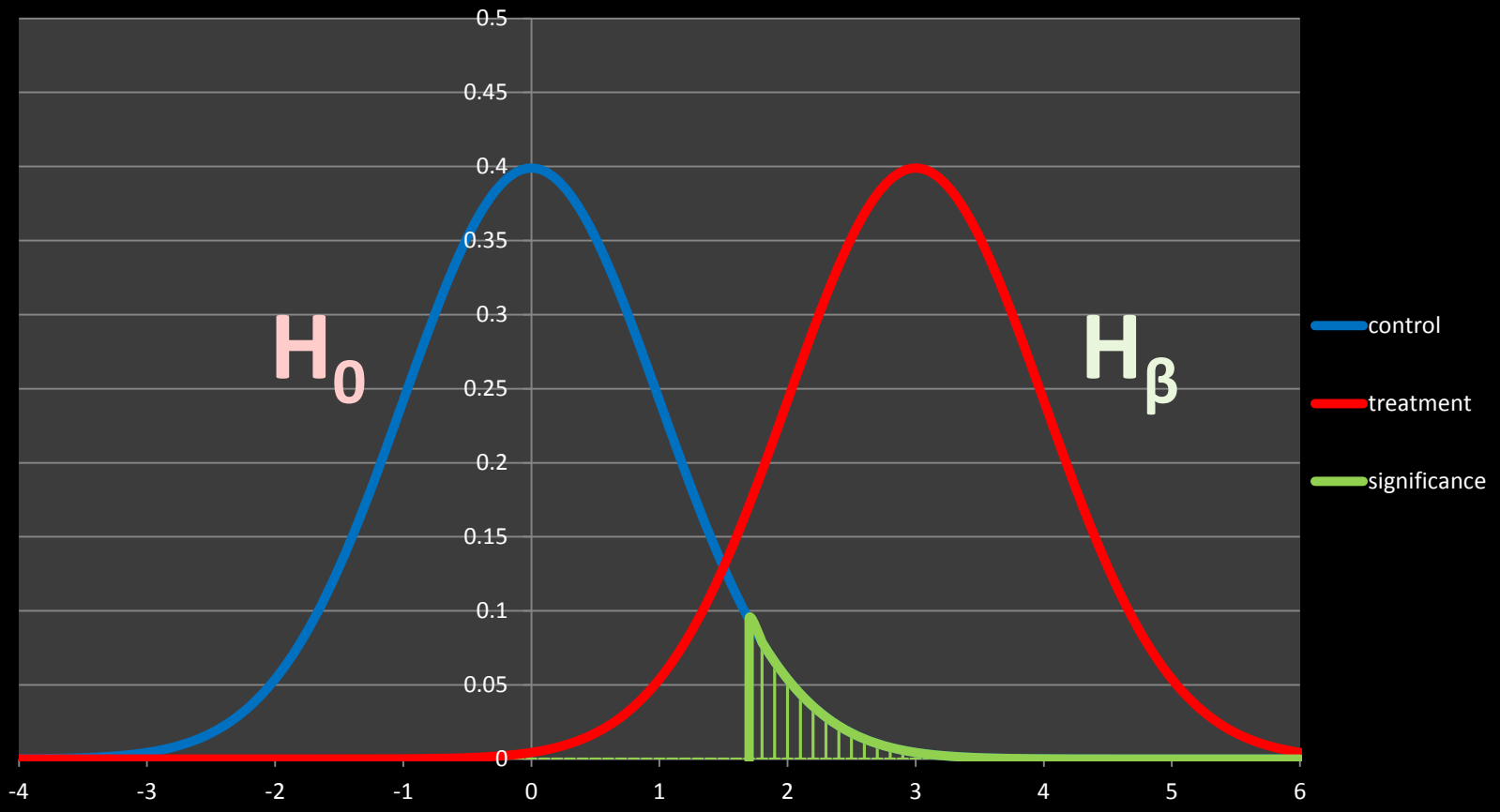
Variance

- There is sometimes very little we can do to reduce the noise
- The underlying variance is what it is
- We can try to "absorb" variance:
  - using a baseline
  - controlling for other variables
    - In practice, controlling for other variables (besides the baseline outcome) buys you very little

Power: main ingredients

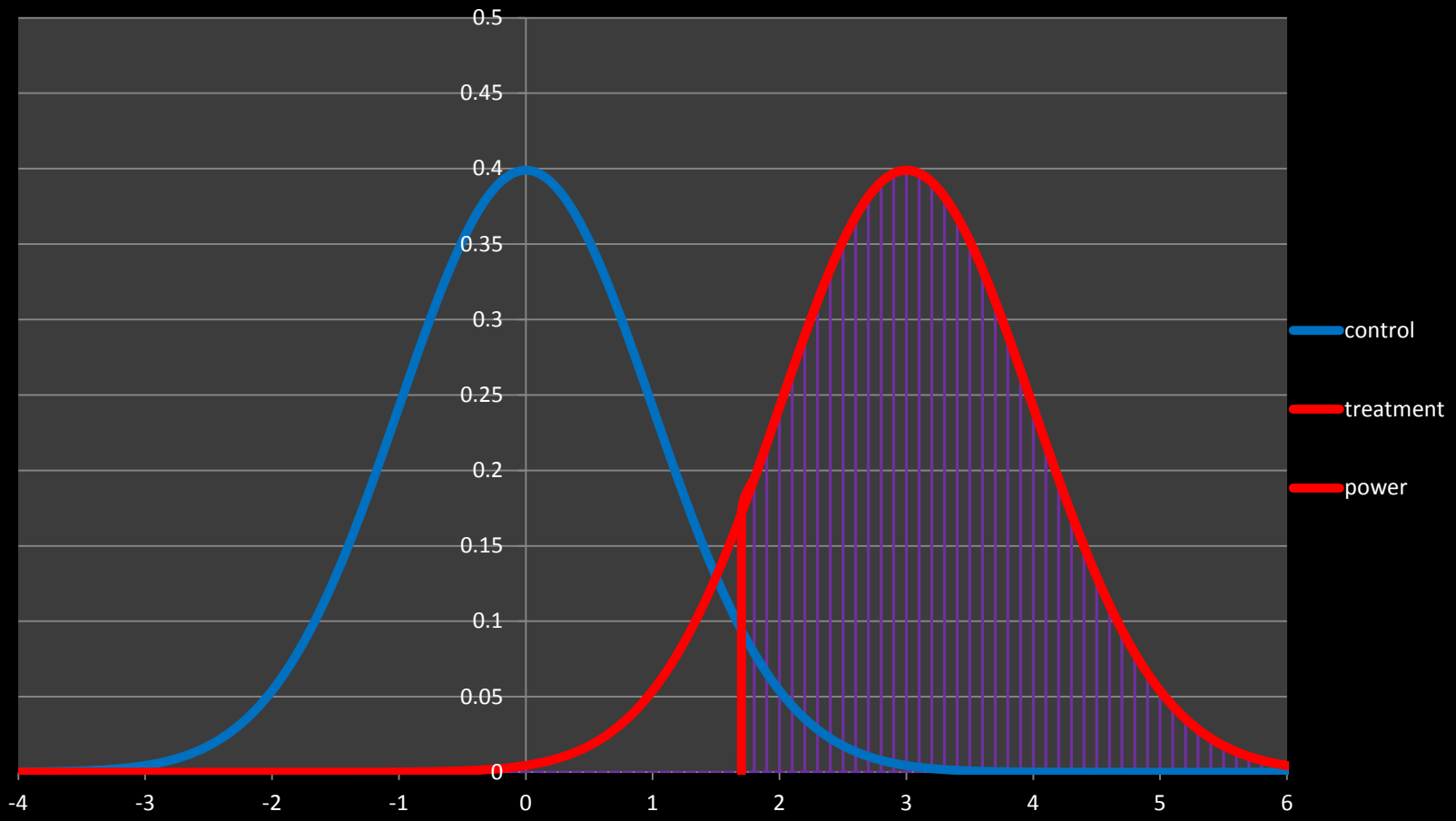
1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

Sample split: 50% C, 50% T



Equal split gives distributions that are the same "fatness"

Power: 91%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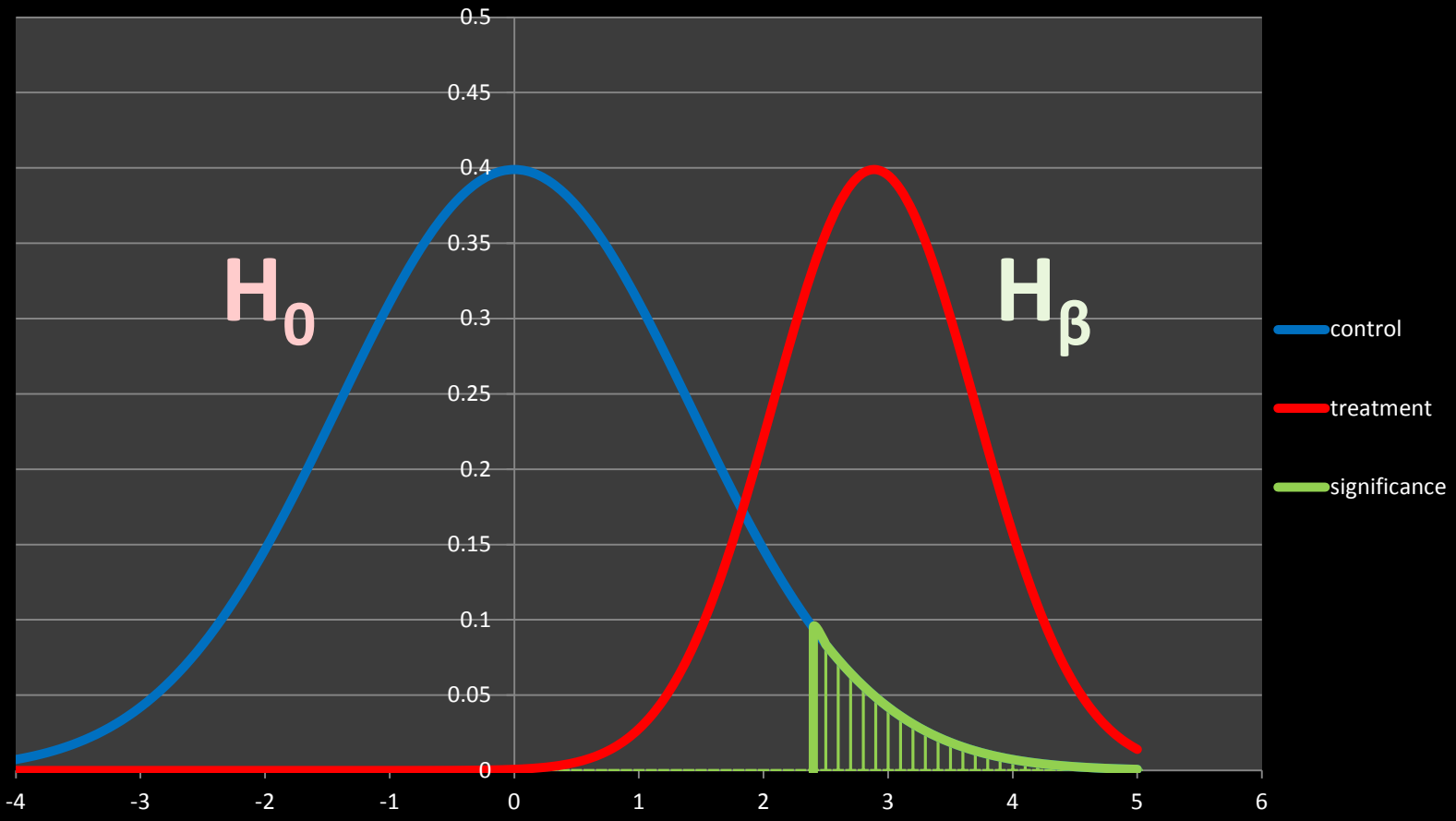
power

If it's not 50-50 split?

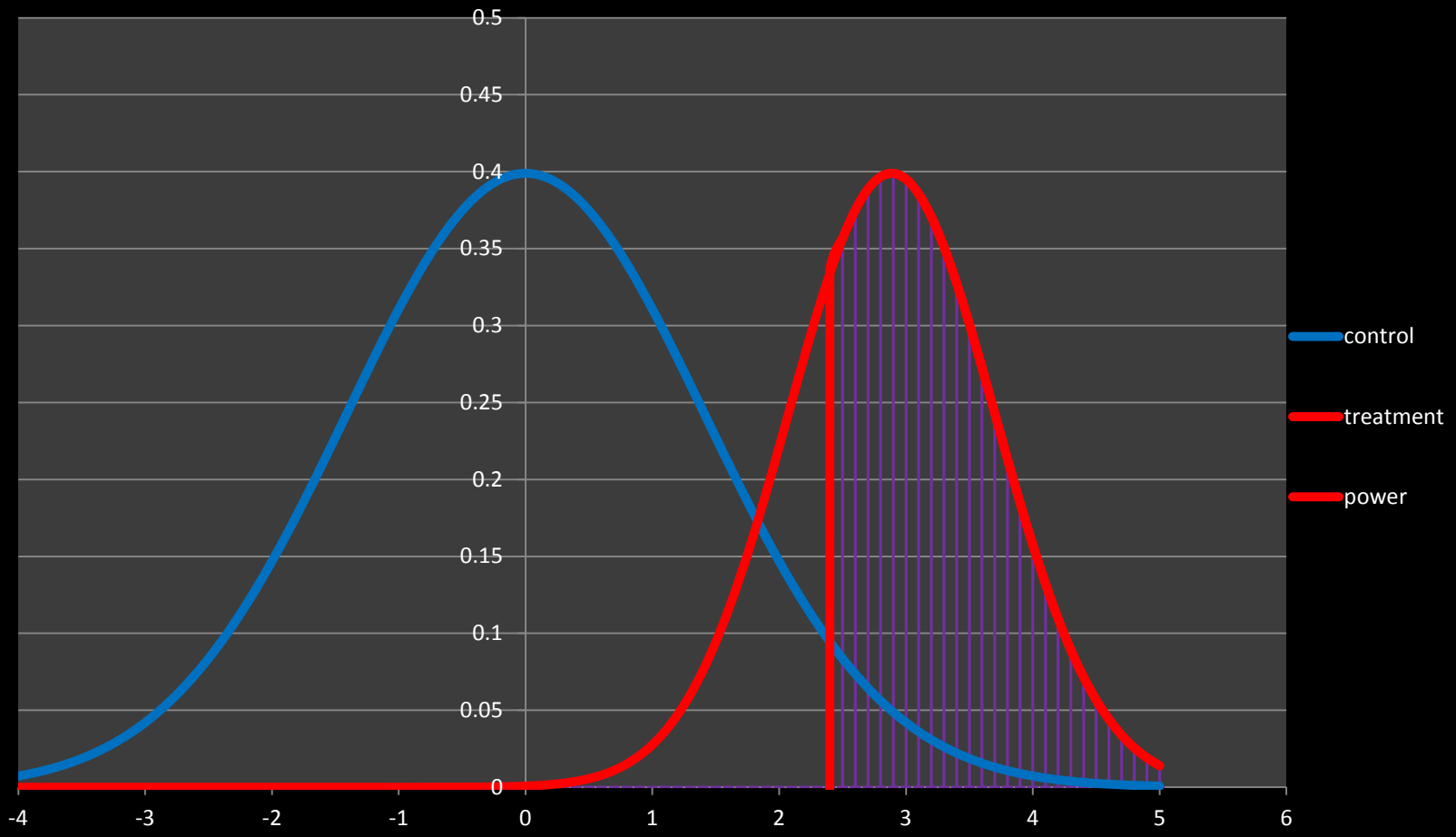
- What happens to the relative fatness if the split is not 50-50.
- Say 25-75?

Sample split: 25% C, 75% T



Uneven distributions, not efficient, i.e. less power

Power: 83%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power

Allocation to T v C

$$sd(X_1 - X_2) = \sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}$$

$$sd(X_1 - X_2) = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{\frac{2}{2}} = 1$$

$$sd(X_1 - X_2) = \sqrt{\frac{1}{3} + \frac{1}{1}} = \sqrt{\frac{4}{3}} = 1.15$$

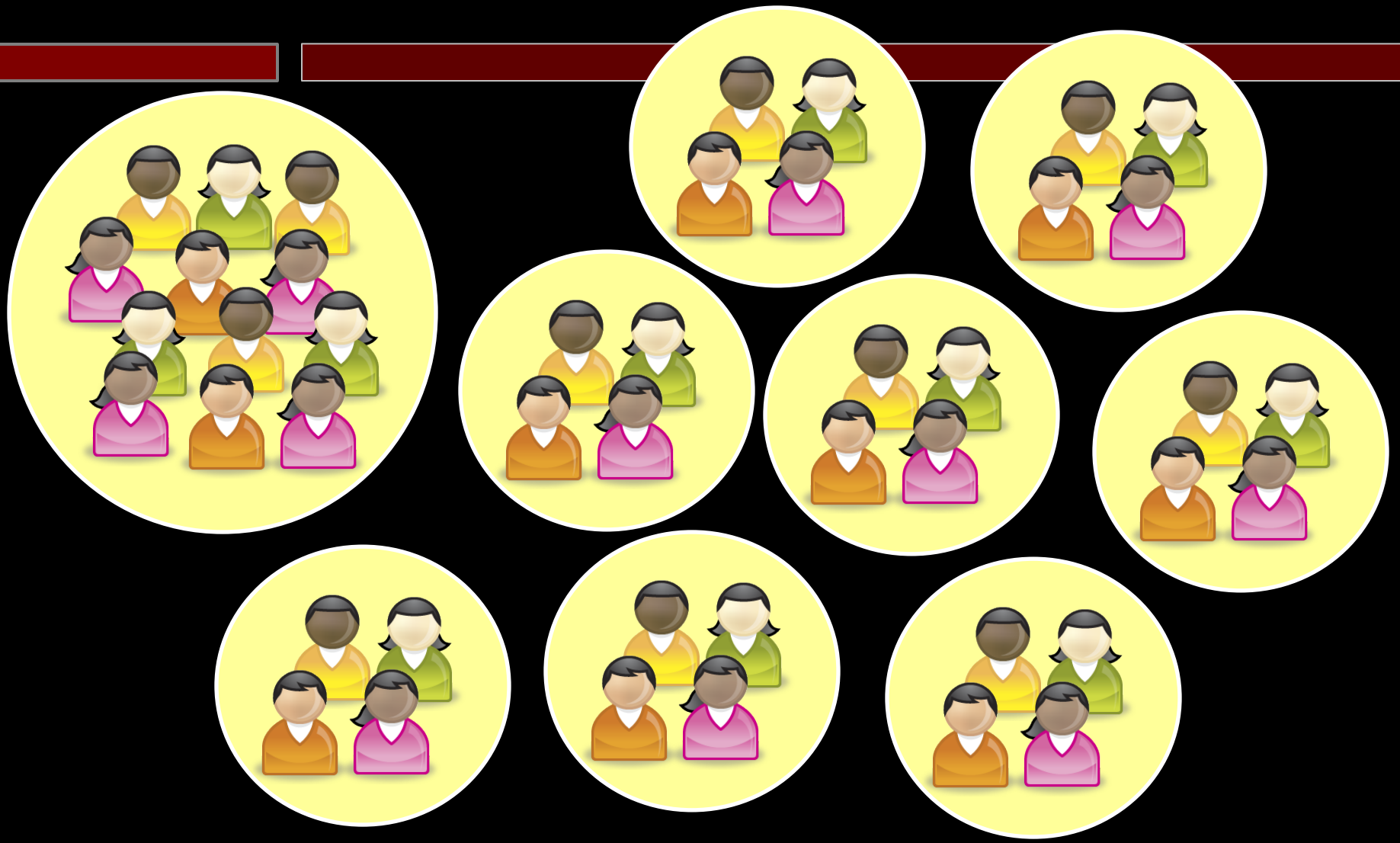
Power: main ingredients

1. Effect Size
2. Sample Size
3. Variance
4. Proportion of sample in T vs. C
5. Clustering

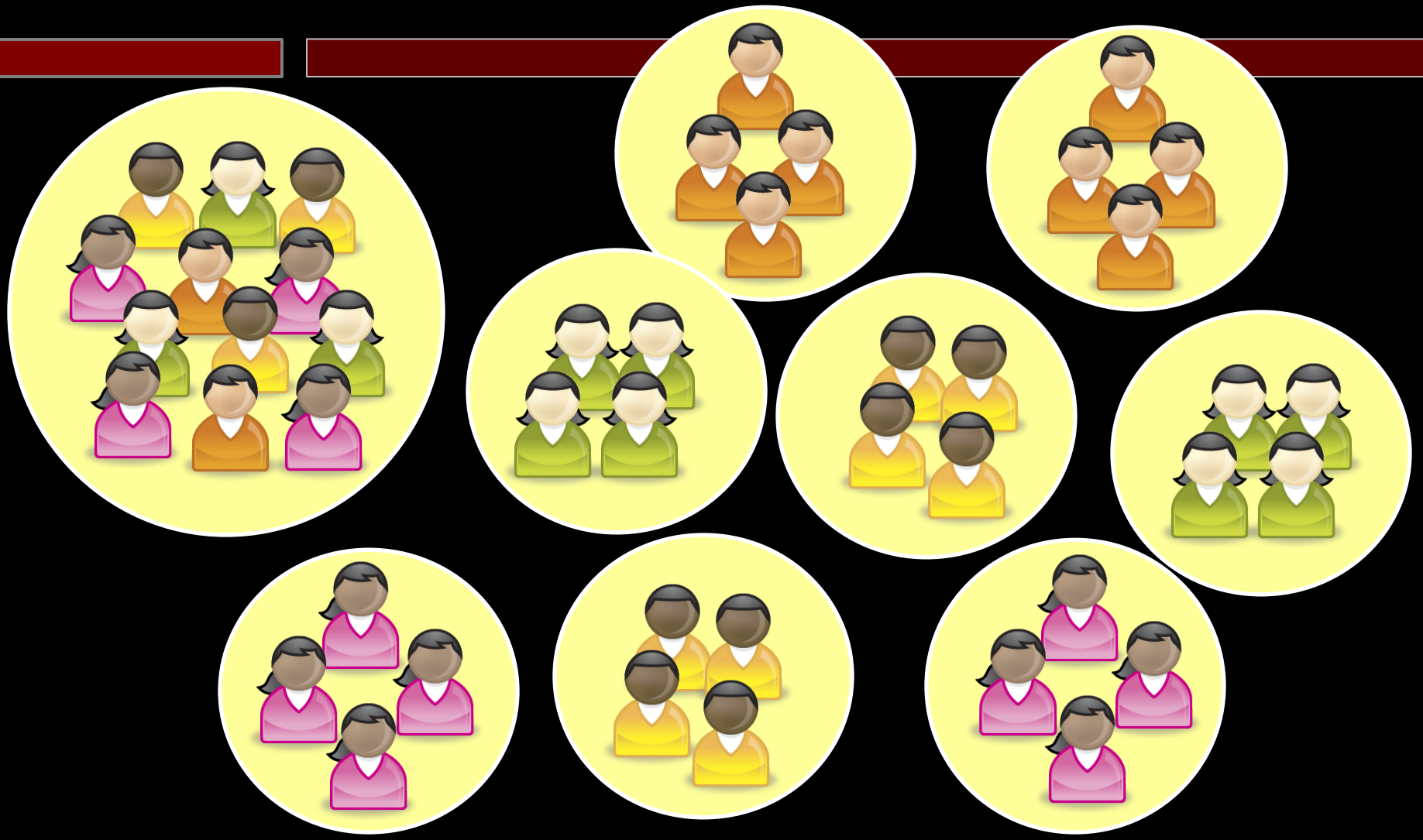
Clustered design: intuition

- You want to know how close the upcoming national elections will be
- Method 1: Randomly select 50 people from entire Indian population
- Method 2: Randomly select 5 families, and ask ten members of each family their opinion

Low intra-cluster correlation (Rho)



HIGH intra-cluster correlation (rho)



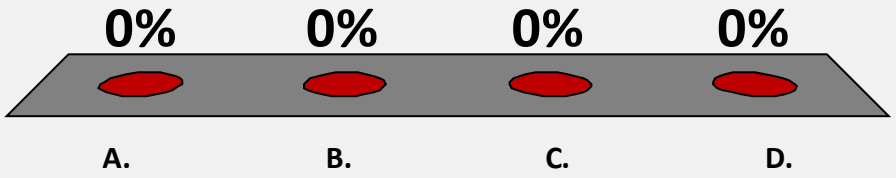
All uneducated people live in one village. People with only primary education live in another. College grads live in a third, etc. Rho on education will be..

A. High

B. Low

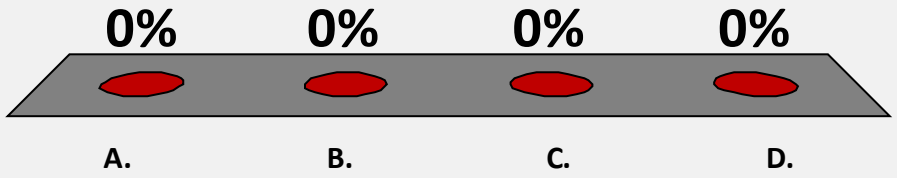
C. No effect on rho

D. Don't know

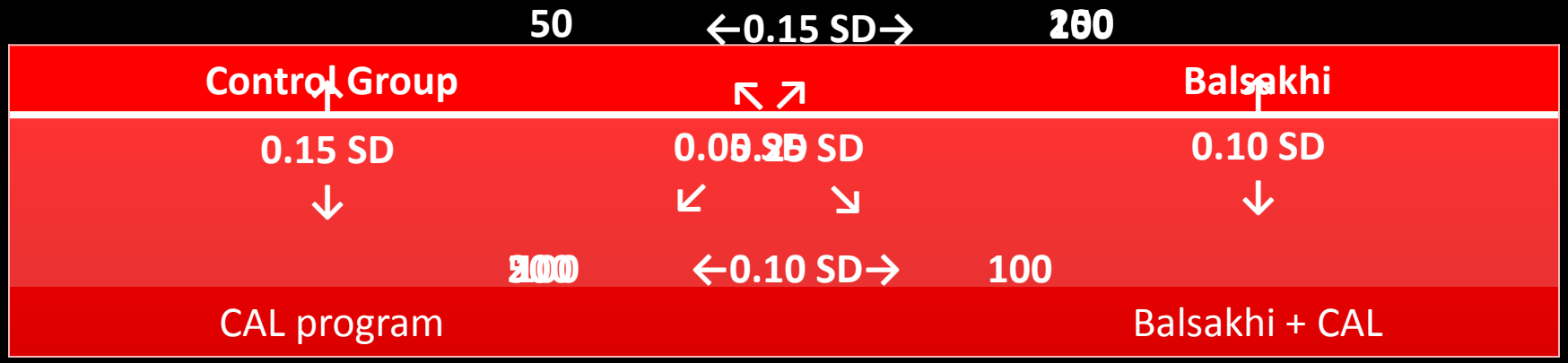


If rho is high, what is a more efficient way of increasing power?

A. Include more clusters in the sample
B. Include more people in clusters
C. Both
D. Don't know



Testing multiple treatments



END!